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STIMULUS SYNTHESIS IN NORMAL AND RETARDED SUBJECTS.

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EXPERIMENTS WERE PERFORMED TO DETERMINE HOW MENTALLY RETARDED AND NORMAL SUBJECTS SYNTHESIZE STIMULI WITH RESPONSES. ALSO, RELATIONSHIPS AMONG THE STIMULUS SYNTHESIS PROCESS, INTELLIGENCE, AND RATE OF LEARNING WERE STUDIED. DISCRIMINATION TASKS WERE USED WHICH REQUIRED SUBJECTS, NORMAL (COLLEGE, HIGH SCHOOL, AND GRADE SCHOOL ENROLLEES) AND MENTALLY RETARDED, TO USE FOUR SEPARATE CUES TO CLASSIFY PICTURES. THE ORDER AND METHODS OF USING THESE CUES WERE THEN STUDIED. FIFTEEN MATHEMATICAL MODELS WERE FORMULATED AND EVALUATED TO DETERMINE THEIR ABILITY TO PREDICT STIMULUS SYNTHESIS PATTERNS. RESULTS RELATED TO LEARNING RATE WERE NOT CONCLUSIVE. THE STIMULUS SYNTHESIS PROCESSES OF MENTALLY RETARDED SUBJECTS DIFFERED FROM THOSE WHO WERE NORMAL. (WB)

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Cooperative Research Project No. 1575

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Fort Worth, Texas

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INTRODUCTION

Historically, the theories of how organisms utilize sensory information have taken two broad paths. The first was concerned with the study of the elements of sensory experience and how they combine via associational processes to build ideas, percepts, and images. The second viewed the organism as endowed with certain native perceptual integrating processes. It was concerned not with the elements that make up a percept, but with the wholes of phenomenal experience.

The antecedents of modern day association theory started in two countries. In England, British empiricism, beginning with Hobbes, Locke, Berkeley and others and ending in the associationism of James and J. S. Mill, made a great impact on western thinking (Boring, 1957). These English scholars emphasized the building of percepts from sensory inputs. They rejected the notion of innate ideas, animal spirits, and some of the mystical explanations of perception that were current in that day. In Germany a great deal of study was devoted to the sense organs. The emphasis of both these groups was on the elements assumed to make up the active mind. The British were more concerned with philosophy or theory, while German scholars emphasized stimulus attributes and the properties of nerve fibers. These efforts converged in the work of Wilhelm Wundt.

There existed in Germany at about the same time another school of thought which was working on quite different assumptions. What we might now call the holistic approach had its genesis in the works of men such as Kant, Hering, Stumpf, and Müller, to name only a few. These men viewed the organism as having certain "givens." For example, the ability to perceive depth was assumed to be a faculty an organism had at birth. For these scholars, sensory information served only to

activate the native unifying principles of the brain. These ideas were carried on in Gestalt psychology.

Gestalt psychology has not changed its position very much in the last fifty years. The rise of Gestaltism is partially explainable as a reaction to the atomistic associationism of Wundt and his students. It is the Gestaltists' argument that parts or elements of perception are only obtainable after the whole has been perceived. To study elements, then, is to study artifacts of the basic process. Gestalt psychology can handle a wide range of perceptual phenomena, but it leaves a broad area of behavior unaccounted for, particularly in the area of learning.

Accepting the Gestalt doctrine nearly precludes the necessity for experimentation. By definition, any type of analysis involves breaking up the wholes into artificial parts. The burden of experimentation has been carried on in the tradition of associationism. It is here that the methodological tools are available. Much of the work in this area has been confined to the studies of one stimulus parameter at a time. In this way it has been possible to determine the kinds of physical energies which the organism can detect and the limits of this detection ability.

Although a cleavage between the divergent approaches of the associationists and Gestaltists to the problem of stimulus synthesis and the use of sensory information still exists, there is some convergence of the different approaches. Notable among the attempts at convergence is the work of Donald Hebb (1949). Hebb has attempted to integrate contemporary perception and learning theory in a physiological theory. He argues that percepts are the product of learning. The only native ability an organism has at birth is the ability to distinguish figure from

ground. He has accounted for the integration of stimuli by postulating reverberating circuits in the brain. Certain patterns of stimulation are described as giving rise to the firing of specified circuits of neurons. The percept is dependent on the particular circuits aroused to action.

A somewhat broader approach has been utilized by some individuals in attempts to find the various stimulus parameters which will account for the variance of behavior (see: Arnoult, 1960; Attneave, 1957; Brunswik, 1940; Gibson, 1957). These studies are mostly concerned with defining the nature of the stimulus and the extent to which it can account for the behavior under study. The work of Egon Brunswik is most relevant to the present approach. Together with Tolman (Tolman and Brunswik, 1935), Brunswik presented an article on the causal texture of the environment. It was their contention that percepts were never absolute, only probable. Brunswik (1943, 1944, 1956) presented this thesis under the name of "probabilistic functionalism."

The organism is described by Brunswik as perceiving numerous cues from its environment. These acquire probability values with respect to their ability to arouse the "correct" percept. A correct percept is defined as one which enables the organism to function more adequately in its environment. This is, of course, a learning process. It is by responding to the percept aroused by assessing cues in a given manner that the organism obtains feedback about the appropriateness of its response. If the response is appropriate for the environmental adjustment needed, the probability values assigned the cues will remain about the same. However, if the response does not enhance adjustment, another weighting of the cues is likely. Brunswik's approach is molar in that he has attempted to explain how all sensory input can be combined to give rise to one percept. Yet, his

approach is molecular from the standpoint that he views perception as the product of the multitude of information the organism receives.

The intriguing aspect of Brunswik's formulations lies in the assumed probabilistic nature of the environmental cues. The model not only poses the research problem of discovering which cues are most highly correlated with a percept (Brunswik, 1940), but it lends itself to the contemporary interest in probability learning. Stochastic models of learning have in recent years been given a great deal of attention (see: Bush and Mosteller, 1955; Estes and Burke, 1953). These models have met with varying degrees of success in predicting the form of a learning curve for a given stimulus situation. The behavior of animals and humans in many types of probability tasks has been studied extensively (see Hilgard, 1956, Chap. 11). Research surrounding these models has not, however, attempted to assess learning when multiple cues are presented.

Seldom, if ever, can behavior be accounted for by the study of one stimulus parameter. There are numerous potential stimuli acting on the organism at any given moment. These include all the proprioceptive stimuli acting from within the organism and all the extroceptive stimuli originating in the environment and conducted to the organism via the sensory processes. Proprioceptive stimuli include the biochemical imbalances and the homeostatic mechanisms, kinesthetic stimuli, and vestibular stimuli, while potential extroceptive stimuli consist of any energy change in the environment the sense organs are capable of relaying to the central nervous system. Taken together, the sources of stimulation are numerous. Assuming that a large number of potential stimuli are filtered out before they arrive at the point where they are instrumental in determining a response, there must still be substantial amounts of information to assess and synthesize before a response is

made. The general purpose of the experiments presented in this report is to examine the process of stimulus synthesis. The term stimulus synthesis is used to describe the operations which occur when an organism combines and integrates the information (cues) it receives to effect a response. Quite obviously, the information used to arrive at a response must be evaluated. Some cues would be weighted rather strongly, others weakly, and some at values in between. Terms such as learning, perception, discrimination, mediation, perceptual learning, and decision making, describe psychological processes that stimulus synthesis is intended to include. It is not the intent of the present report to describe the synthesis process into these terms, but rather it is hoped these studies will be applicable to the whole area the terms include.

Elam (1962) reported several experiments in which attempts were made to describe the process human subjects used to synthesize stimulus cues in a learning task. Subjects viewed a series of slides having from one to four stimulus cues. Each of the cues had 20 or 21 variations and each variation had a predetermined probability of being reinforced for two categories of response, e.g., subjects could respond "left" or "right," "X" or "Y," "up" or "down," etc. Four hundred twenty such slides were used for each treatment group. Subjects viewed the slides, indicated their answers, and then were told which answer was "correct." Elam's experiments investigated three hypotheses of stimulus synthesis.

1. Linear Hypothesis. This hypothesis stated that equal weight was assigned each cue and that synthesis was the adding together of each of these cues.

2. Multiplicative Hypothesis. This hypothesis stated that the high or low cue variations would be given more weight in the synthesis process than those cue variations which tended toward the middle values.

3. Log Hypothesis. This hypothesis stated that the middle cue variations would be given more weight at the expense of the extreme values.

In addition, the analyses included controls to determine whether or not subjects were in fact synthesizing the cues, i.e., using more than one cue in solving the problem.

The data were analyzed by first calculating theoretical scores which described functions the three hypotheses tested would predict. These values were then correlated with performance scores obtained by experimentation.

The analyses of two experiments indicated that the multiplicative and linear hypotheses came closest to describing the process although in one case, the best predictor was a single cue, i.e., the subject apparently ignored three of the cues and answered totally on the basis of a fourth cue.

One criticism of Elam's research might be that it ignores or obscures individual differences in subjects. Taking cognizance of this fact, an unpublished study was conducted for a two-cue problem on a group of college sophomores. A measure of response strength was obtained on each trial. This was accomplished by having each of 100 subjects "bet" on the likelihood of any slide being an "X" or "Y." Each subject's score was then correlated with predictions from the various hypotheses over every block of 42 trials. There were no cases in which a subject's performance was consistently correlated highly with a particular hypothesis. Rather, the trend appeared to be somewhat the same for all subjects. That is, one hypothesis would be employed, then discarded, and another adopted and discarded, etc. On the basis of these data, the additive, multiplicative, and square root functions appeared strongest.

The present report is a continuation of these investigations.

STUDY NO. 1

STIMULUS SYNTHESIS IN A TWO CHOICE DISCRIMINATION

This experiment represents an effort to obtain a very general description of the method by which subjects integrate stimulus cues. In particular an attempt was made to determine whether normal and retarded subjects differ in their method of synthesis.

Basically, the procedure was one of giving the subject a discrimination which could be solved (at least in a probabilistic sense) on the basis of four independent stimulus cues. The object of the analysis was to determine how the separate cues were perceptually integrated into a unitary response. This was done by obtaining estimates of how the subjects were responding to the individual cues. Then by combining this information in three different ways (Linear, Multiplicative, and Square Root Solutions) the solutions were compared with the actual results of the experiment.

METHOD

Stimuli

Six basic groups of stimuli were produced for this study. These are referred to as the Random, Selected, Correlated, B+A, Single and Constant Stimulus Groups. With exception of the Constant Stimuli, these groups of stimuli were given to separate groups of subjects and constituted the sole difference in the treatments. Consequently, reference is occasionally made to "Selected Stimulus Treatment Group"

or "Random Group," etc. The procedure used in generating these stimulus populations is described below.

Random Stimuli. This group consisted of 400 pictures; an example of which is shown in Figure 1. On the basis of previous experimentation (Elam, 1962), as



Figure 1. An example of the stimuli used.

well as on analytical considerations, it was desired that each picture should contain four stimulus cues which related to the solution of the tests for which the stimuli were produced. It was also determined that each cue would show 20 variations on its defined continuum within the total population of 400 pictures and that each variation would occur an equal number of times. The cues in the present case were the position of the arms, legs, and the head of the model, and the length of the bar over the model's head. Thus, each position of the arms of the 20 total positions shown occurred 20 times during the 400 pictures.

In a preliminary experiment it was insured that the 20 variations of each cue were linear in their subjective displacement from one another. This was

accomplished by having six subjects evaluate the four cues on a 0 to 100 scale. Each variation was tachistoscopically presented (2 second exposures) 20 times and the subjects were required to write down a numerical equivalent to the variation shown. The limits of the scale were described to the subjects prior to testing. With the head, for example, they were told that when this member was turned fully to the left the correct value would be zero, but would be 100 when turned fully to the right. The variations were randomly presented. The means for the 20 presentations of each variation of the four cues for all six subjects were then calculated. These are shown in Figure 2. Since these functions are very close to being linear, it can be assumed that the perception of change was also approximately linear with the physical variation of the cues.

In generating the 400 pictures of the Random Stimulus Group, it was the purpose to delineate two subpopulations of stimuli. It is convenient to refer to these as the X and Y stimuli. The X population can be defined as consisting of 200 pictures of the figure in leotards with the length of the bar and the position of the arms, legs, and head having the frequency functions shown in Figure 3. Referring to the upper left figure, it will be seen that of the 200 X stimuli, only one showed the legs in the position of being together (Position 20). There were five pictures with the legs in the thirteenth position and 19 with the legs fully apart (Position 1). Similarly, there was only one picture of the 200 X stimuli in which the arms were in the top-most position, but 19 of them in the lowest position. This is seen in the lower right figure. The other variations followed similar frequency patterns as can be appreciated from the other curves of Figure 3.

Figure 2. Results of subject judgments of the four cue continua based upon a 0 - 100 scale.

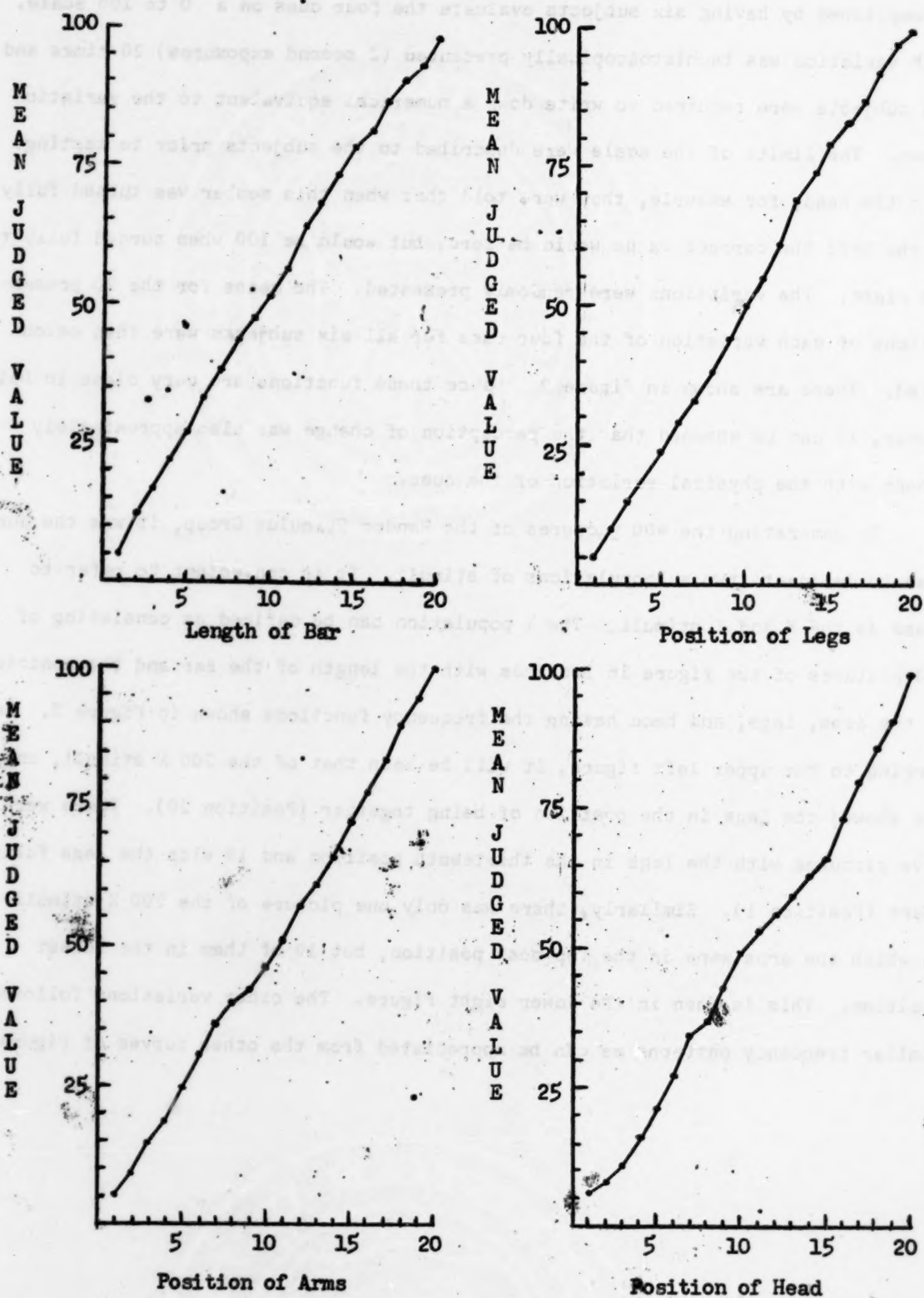
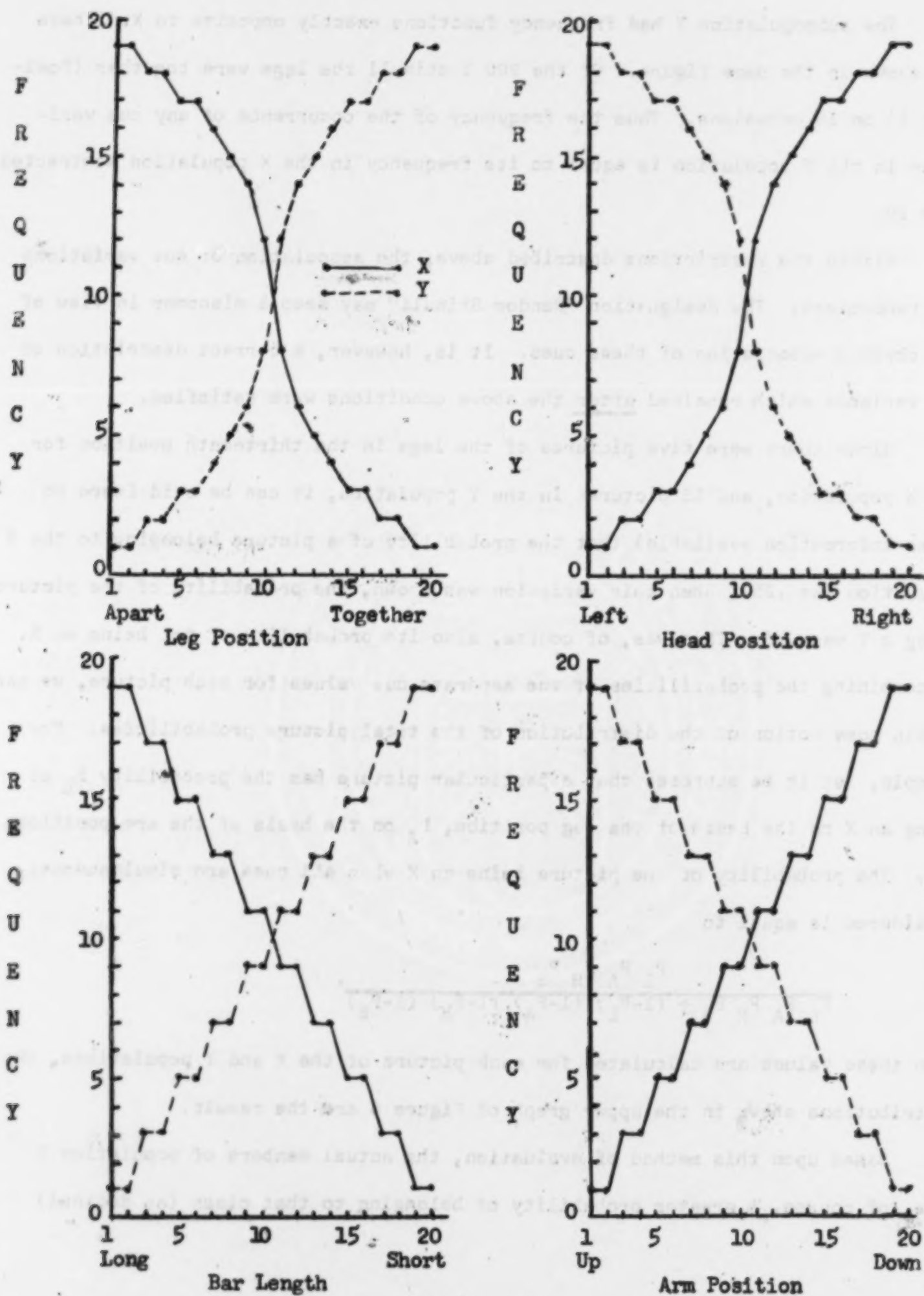


Figure 3. Frequency of occurrence of each cue variation in the X and Y sub-population of stimuli.



The subpopulation Y had frequency functions exactly opposite to X. These are shown in the same figure. Of the 200 Y stimuli the legs were together (Position 1) on 19 occasions. Thus the frequency of the occurrence of any cue variation in the Y population is equal to its frequency in the X population subtracted from 20.

Within the restrictions described above, the association of cue variations was randomized. The designation "Random Stimuli" may seem a misnomer in view of the obvious association of these cues. It is, however, a correct description of the variance which remained after the above conditions were satisfied.

Since there were five pictures of the legs in the thirteenth position for the X population, and 15 pictures in the Y population, it can be said (were no other information available) that the probability of a picture belonging to the X population was .25. When this variation was shown, the probability of the picture being a Y was .75. This was, of course, also its probability of not being an X. By combining the probabilities of the separate cue values for each picture, we can obtain some notion of the distribution of the total picture probabilities. For example, let it be supposed that a particular picture has the probability P_L of being an X on the basis of the leg position, P_A on the basis of the arm position, etc. The probability of the picture being an X when all cues are simultaneously considered is equal to

$$\frac{P_L P_A P_H P_B}{P_L P_A P_H P_B + (1-P_L)(1-P_A)(1-P_H)(1-P_B)}$$

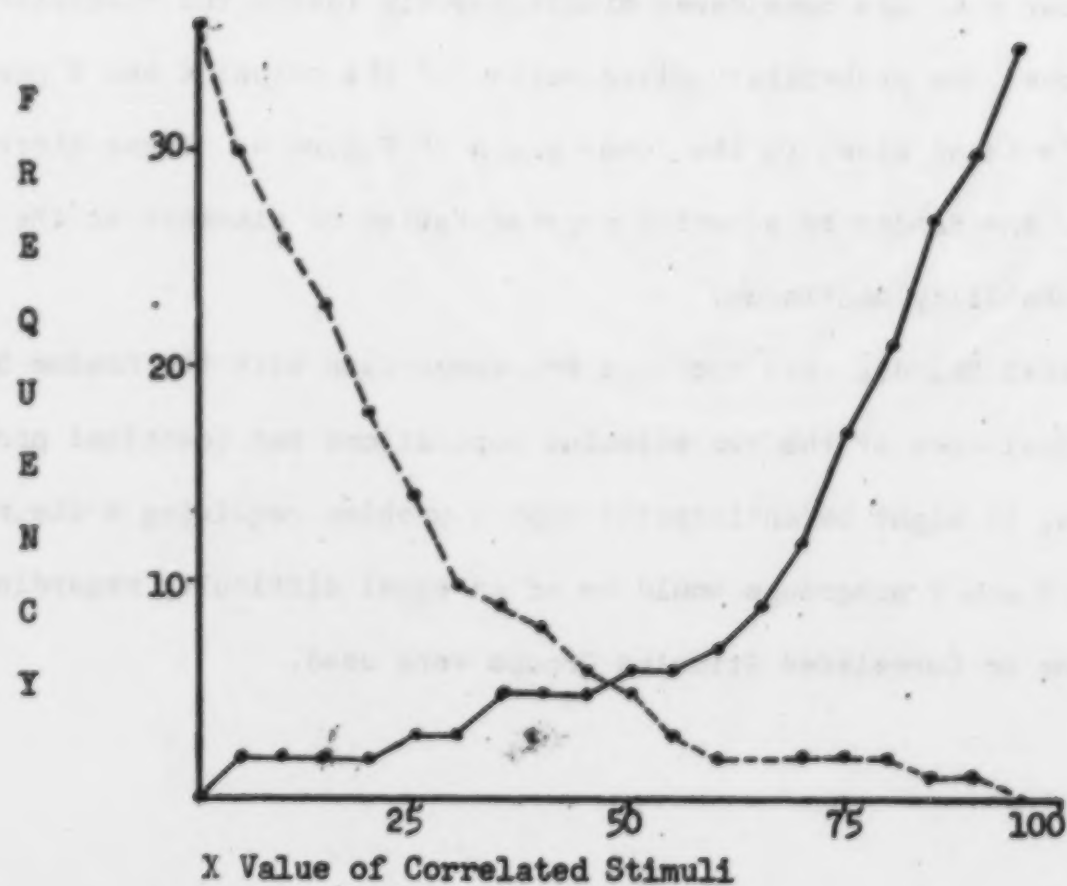
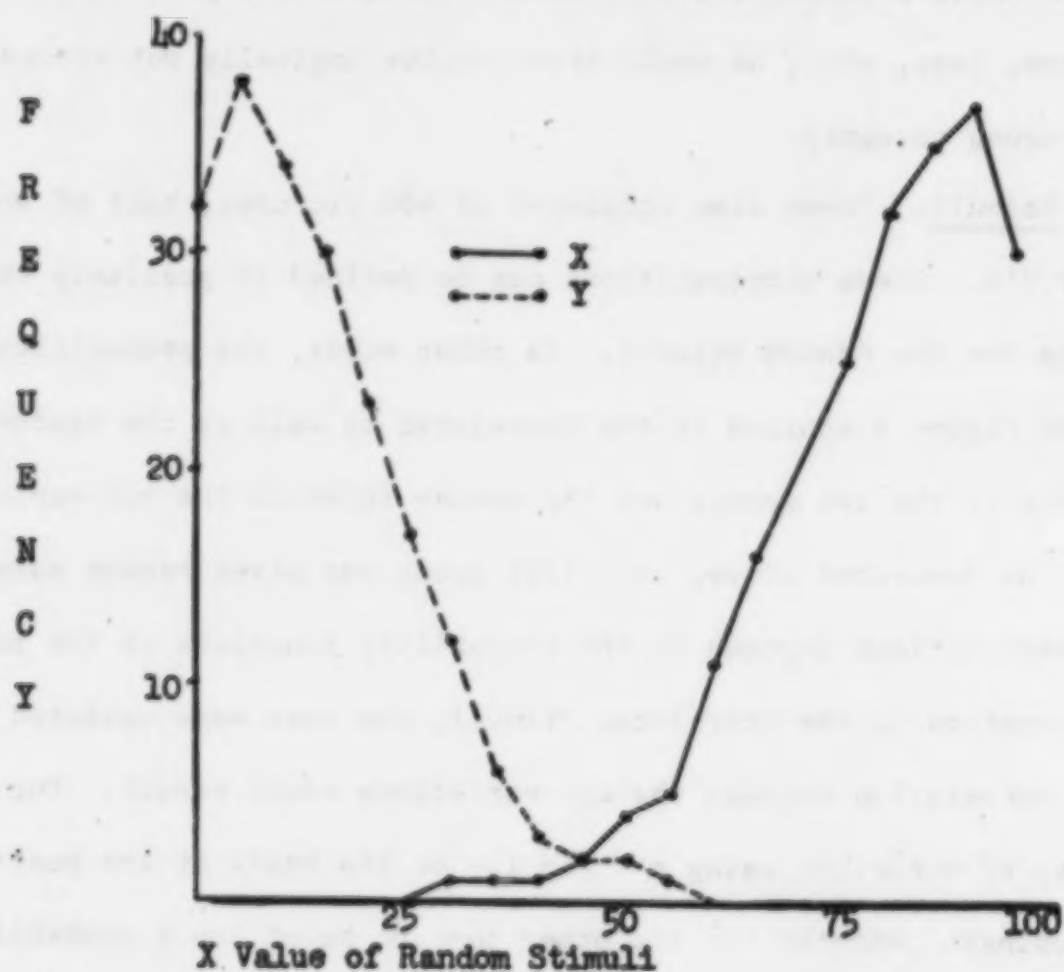
When these values are calculated for each picture of the X and Y populations, the distributions shown in the upper graph of Figure 4 are the result.

Based upon this method of evaluation, the actual members of population X have, of course, a greater probability of belonging to that class (as defined)

Figure 4. Frequency distribution of the X value

$$P_X = \frac{P_B P_L P_A P_H}{P_B P_L P_A P_H + (1-P_B)(1-P_L)(1-P_A)(1-P_H)}$$

of the X and Y subpopulation of the Random and Correlated Stimuli.



above) than do the actual members of population Y. There is, however, some overlap. Thus, if one knows a population only on the basis of its probability distribution for the arms, legs, etc., he would often, quite logically but erroneously, assign it to the wrong category.

Correlated Stimuli. These also consisted of 400 pictures, half of which were X's and half Y's. These subpopulations can be defined in precisely the same manner as was done for the Random Stimuli. In other words, the probability distribution shown in Figure 3 applies to the Correlated as well as the Random Stimuli. The difference in the two groups was the manner in which the cue variations were associated. As described above, the first group was given random association (except for the restrictions imposed by the probability functions of the individual cues). In the formation of the Correlated Stimuli, the cues were selected so that a high degree of correlation between the cue variations would result. For example, if the probability of a picture being a Y was low on the basis of arm position, there was an inordinate tendency for the other cues to be of low Y probability.

When the four cues are considered simultaneously (using the Multiplicative Formula given above) the probability distribution of the actual X and Y populations of being X's is as shown in the lower graph of Figure 4. These distributions differ from the Random by a better representation of elements at the extremes of the probability continuum.

The Correlated Stimuli were produced for comparison with the Random Stimuli. Since the individual cues of the two stimulus populations had identical probability distributions, it might be anticipated that a problem requiring a discrimination between the X and Y subgroups would be of an equal difficulty regardless of whether the Random or Correlated Stimulus Groups were used.

A second hypothesis can be entertained, however. Correlation of stimulus elements is simply a mathematical description of information redundancy. If the cues are perfectly correlated, the presence of multiple cues neither reinforce nor detract from the quality of information given by a single cue. While this statement is not altogether true within the present context (since the cues action in concert reflect the probabilities more accurately than when acting alone) this is not known to the subject. One might consequently infer that the rate of learning using the Correlated Stimuli will be somewhat less than when the Random Stimuli are used.

The opposing point of view could also be presented, however. In one sense the subject given the Correlated Stimuli is relieved of the necessity of integrating the cues since each is redundant to the other. The subject given the Random Stimuli is presented with heterogeneous probability estimates from the four information sources which he must synthesize if he is to maximize his response strategy. In the Correlated Stimuli much of this synthesis is done for him through the association of cues.

Selected Stimuli. This group also consisted of 200 X and 200 Y pictures with the four cues having the probability distributions shown in Figure 3. It differed from the Correlated and Random Groups in that the association of cues was selected to provide the greatest possible variation or conflict of information. From this standpoint, the Correlated Stimuli presented the greatest probabilistic uniformity between stimuli while the Selected Stimuli involved the least coincidence of this kind. The Random Stimuli were intermediate between these two groups. Another way of stating this relationship is to say that of the variance remaining after the individual stimulus probability functions were satisfied,

one was positively correlated (Correlated Stimuli), one was negatively correlated (Selected Stimuli), and one was uncorrelated (Random Stimuli).

Since the studies were concerned with the problem of synthesis or probability integration the Selected Stimuli were considered to be the best for this purpose. It was for this reason that the B+A and Single Stimuli Groups were designed to correspond to the Selected Stimuli in those cues which were relevant. A second advantage to the use of the Selected Stimuli arose from the type of analysis performed. One assumption of the correlation coefficient is homoscedasticity. Taken in concert, the X and Y subpopulations of both the Correlated and the Random Groups tend to produce bimodal probability distributions. This is much less true of the Selected Stimuli. The probability distributions for all four cues taken simultaneously within the Selected Stimuli are shown in the upper graph of Figure 5.

B+A Stimuli. The X and Y subpopulations of this group were identical with those of the Selected Stimuli Group except that the position of the legs and head, while variable, were irrelevant. In other words, the distributions shown for legs and head in Figure 3 did not apply to this group of stimuli. The distribution of these variables were constant at the 50% level for both X and Y. The probability distributions of the bar and arms, however, were as shown in Figure 3, and the association as well as the order of presentation of these variables were identical with the Selected Stimuli.

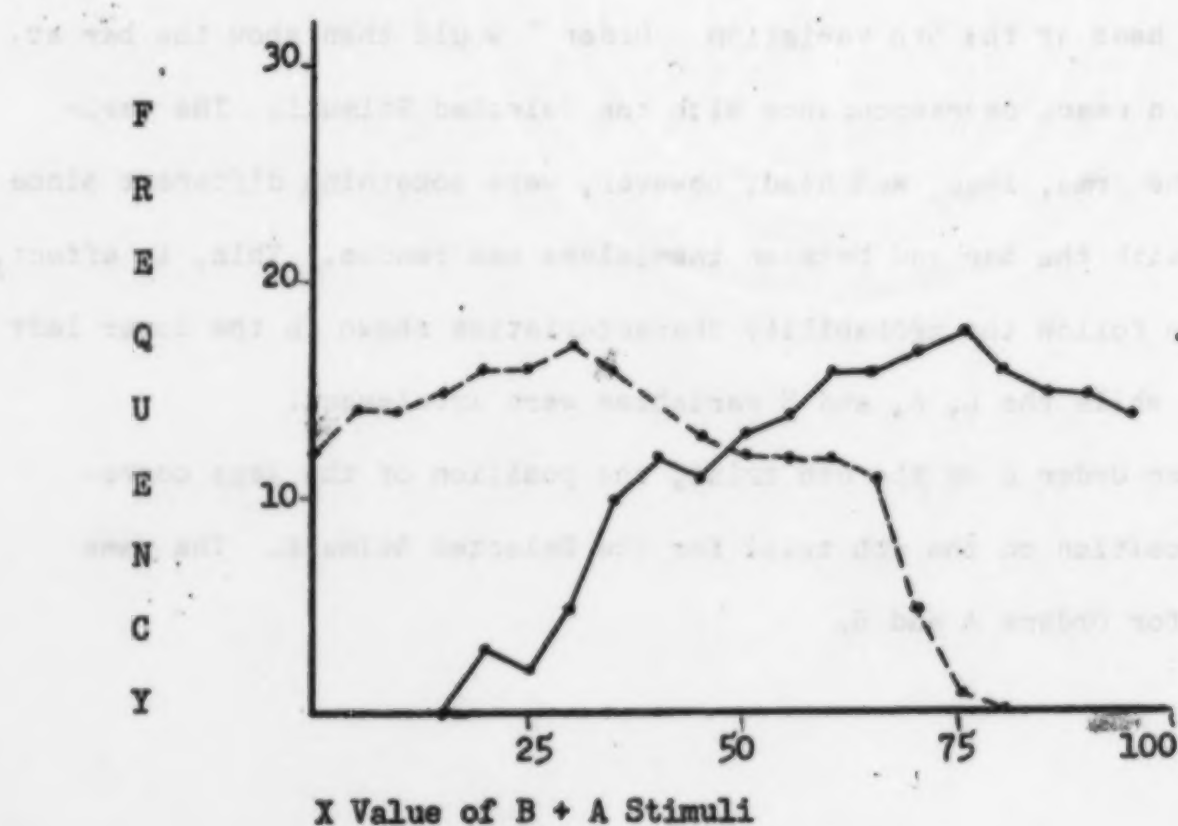
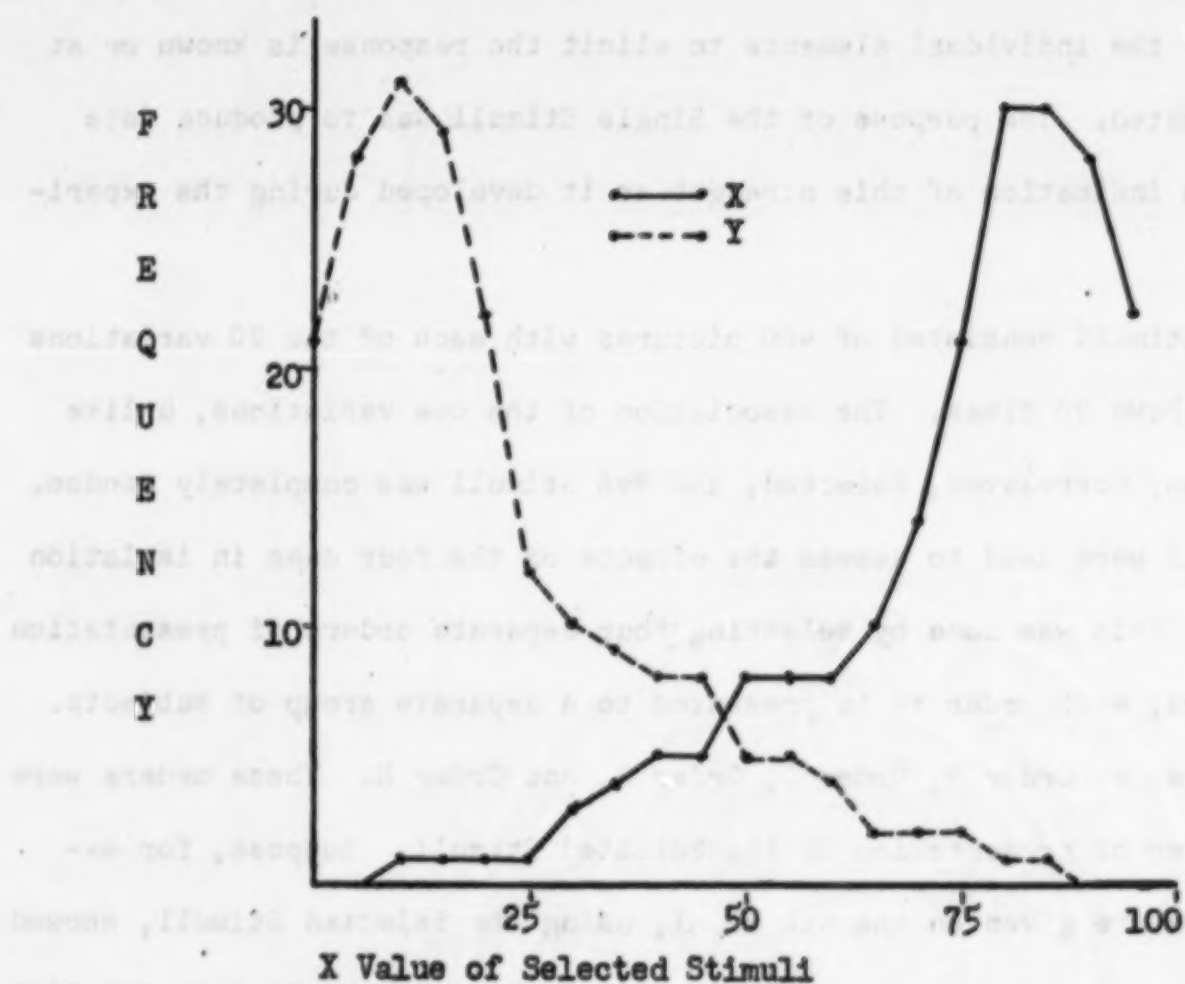
The purpose of this group was as a check on whether the two relevant-variable problems (B+A) obtained the same type of synthesis as would be obtained for the four-variable problem (Selected).

The probability distribution for all cues (relevant and irrelevant) of the B+A Stimulus Group were as shown in the lower graph of Figure 5.

Figure 5. Frequency distribution of the X value

$$P_X = \frac{P_B P_L P_A P_H}{P_B P_L P_A P_H + (1-P_B)(1-P_L)(1-P_A)(1-P_H)}$$

of the X and Y subpopulation of the Selected and B+A Stimuli.



Single Stimuli. Since the discriminative response is based upon some selection of the stimulus cue elements available, the problem of analysis is facilitated if the strength of the individual elements to elicit the response is known or at least can be estimated. The purpose of the Single Stimuli was to produce data that would give an indication of this strength as it developed during the experimental procedures.

The Single Stimuli consisted of 400 pictures with each of the 20 variations of the four cues shown 20 times. The association of the cue variations, unlike those of the Random, Correlated, Selected, and B+A Stimuli was completely random.

These stimuli were used to assess the effects of the four cues in isolation from one another. This was done by selecting four separate orders of presentation of the 400 pictures, each order to be presented to a separate group of subjects. These are designated as Order B, Order L, Order A, and Order H. These orders were based upon the order of presentation of the Selected Stimuli. Suppose, for example, that the picture given on the n th trial, using the Selected Stimuli, showed the bar at the 8th variation, the arms at the 10th variation, the legs at the 16th variation, and the head at the 5th variation. Order B would then show the bar at the 8th variation in exact correspondence with the Selected Stimuli. The variations shown for the arms, legs, and head, however, were something different since their association with the bar and between themselves was random. This, in effect, made the B variable follow the probability characteristics shown in the lower left graph of Figure 3, while the L, A, and H variables were irrelevant.

Similarly, for Order L on the n th trial, the position of the legs corresponded to their position on the n th trial for the Selected Stimuli. The same relationship held for Orders A and H.

In this manner it was possible to obtain an estimate of the strength (probability of its being perceived as an X) for each cue variation for each trial of the Selected Stimuli with a minimum influence from the other cues. The data obtained from the four groups of subjects assigned to these four orders of presentations could consequently be used in the production of the predictive equations used on the results of the group assigned the Selected Stimuli problem.

Constant Stimuli. The purpose of this group of stimuli was identical with that described for the Single Stimuli. Hence, it afforded an alternative method for estimating the strength of the individual stimulus cues in isolation from one another. The basic difference in their use was in the fact that the four orders of the Single Stimuli were given to four separate groups of subjects while the Constant Stimuli were given to all groups. They were interspersed between the other stimuli to which the group had been assigned.

The Constant Stimuli consisted of a number of systematic associations of cues which were combined so that the individual cue probability functions could be assessed. The basic arrangement is shown in Table 1.

Subgroup 1 consisted of 20 pictures. In each of these the legs, arms, and head were constant at the neutral positions (10.5). The length of the bar, however, varied, being at Position 1 for one picture, Position 2 for another picture, etc. Since the neutral positions were likely to produce a 50-50 split in X-Y judgements, any significant variations from the 50% level of response could be attributed to the cue (in this case the length of the bar) which was allowed to vary. In this manner the probability function of the bar alone could be assessed. The same logic applies to the legs, arms, and head on the basis of Subgroups 2, 3, and 4.

Table 1

Description of the Pictures Comprising the Constant Stimuli Group

Subgroup	No.	Bar	Cue Value		
			Legs	Arms	Head
1	20	variable 1-20	constant 10.5	constant 10.5	constant 10.5
2	20	constant 10.5	variable 1-20	constant 10.5	constant 10.5
3	20	constant 10.5	constant 10.5	variable 1-20	constant 10.5
4	20	constant 10.5	constant 10.5	constant 10.5	variable 1-20
5	20	variable 1-20	variable 20-1	constant 10.5	constant 10.5
6	20	variable 1-20	constant 10.5	variable 20-1	constant 10.5
7	20	variable 1-20	constant 10.5	constant 10.5	variable 20-1
8	20	constant 10.5	variable 1-20	variable 20-1	constant 10.5
9	20	constant 10.5	variable 1-20	constant 10.5	variable 20-1
10	20	constant 10.5	constant 10.5	variable 1-20	variable 20-1

Subgroups 5, 6, 7, 8, 9, and 10 were contrived in a manner such that two cues varied simultaneously in complementary but probabilistically opposite directions. One of the pictures in Subgroup 5, for example, might show the bar at a length that had a .25 objective probability of being an X. In the same picture the position of the legs had a .75 probability of being an X. Thus the overall probability

of the picture being an X, based upon any systematic account of the cues, was .50. The same statement applies to all pictures in this subgroup and all succeeding subgroups illustrated in the table.

Response variations (other than chance) were consequently attributable to the greater influence of one cue than the other. These pictures provided a means of assessing weighting factors for use in the analysis of synthesis process.

The remaining 200 pictures of the Constant Stimuli were essentially duplicates of those shown in Table 1 except that the constant values were assigned different positions as a check on the high and low characteristics of the probability functions.

Subjects

The assignment of subjects is shown in Table 2. In all, 282 subjects were used in this experiment. One hundred were from the public high schools, 69 from the public grade schools, and 113 from the State School for the Mentally Retarded in Denton, Texas. A larger number in each of these categories were actually tested, but for various reasons (illness or other interference) some were unable to complete the total schedule and their results were not used in the analysis.

The high school subjects (HS) were from the tenth and eleventh grades and were within the 16 to 17 year age bracket. Generally they were from the upper middle class socioeconomic level, but there were wide variations in this variable. All were volunteers and received a small remuneration (proportional to the number of correct answers given) for their participation.

The grade school subjects (GS) were drawn from the fifth and sixth grades and were in the 10 to 11 year age bracket. They were from the same neighborhood as

Table 2

**Assignment of the Three Subject Sources to the Eight Treatment
Conditions of the Experiment**

Stimulus Treatment	High School	Grade School	Retarded	Total
Random	13	9	0	22
Correlated	18	11	0	29
Selected	16	11	33	60
B+A	12	9	28	49
B	10	7	12	29
L	11	5	14	30
A	9	8	12	29
H	11	9	14	34
Total	100	69	113	282

the high school subjects and of an approximately equal IQ. They also received pay in proportion to performance.

Since it was necessary to rely on the school records for IQ and scholastic attainment and since the method of evaluation differed between institutions, it is difficult to speak with any absolute confidence about the mean or range of this parameter. The average IQ for the high school group was, however, calculated to be approximately 108 and the mean of the grade school group somewhat higher, being approximately 114.

The retarded children (R) were drawn from two groups at the Denton School. These were made up of two levels of development. The classification was based on a system described by Sloan and Berch (1955). The lower development group, Level III, could be described as "Educable." Their social awareness was generally good,

and their motor development was fair to good. There was an adequate communication between the subjects and the testers. These children were capable of maintaining themselves and caring for their immediate needs. It was predicted that they would eventually develop into persons capable of unskilled or semi-skilled employment. Their IQ's were estimated to be between 40 and 60.

The second group (Level IV) was estimated to have IQ's between 50 and 70. They, also, were "Educable," having a potential to learn approximately sixth grade material. Social and communication skills were developed but, of course, not normal. There was little evidence of serious motor defectiveness. This group constituted the highest level admitted to this institution.

For purposes of analysis the retarded groups have been combined. This was done because it was impossible to obtain large enough groups for individual analysis. There were, however, approximately equal numbers from each classification assigned to each treatment condition. These subjects were given candy (M & M chocolate drops) for each correct response.

An effort was also made to keep the ratio of males to females equal for each subject source as applied to each treatment condition. This did not always turn out to be precisely equal, due to the fact that it was necessary to drop some of the subjects due to their inability to complete the experiment or in some cases where the testers inadvertently made an error in administering the tests.

As seen in Table 2, the high school and grade school subjects were assigned to one of eight (Random, Correlated, Selected, B+A, B, L, A, and H) treatment conditions. Assignment was random with respect to these categories. The retarded subjects were not assigned to the Random or Correlated series. This occurred because it had been found by the time that the retarded were scheduled for testing

that the Selected Group of stimuli were superior to the Random and Correlated Groups in usefulness for analysis. Since the number of retarded subjects was limited, it was deemed advisable to concentrate on the better stimuli.

Procedure

There were 400 pictures in each of the Random, Correlated, Selected, and the B+A Stimulus Groups. There were also 400 pictures in the group labeled Single Stimuli from which the orders used for the B, L, A, and H treatments were derived. The Random, Correlated, and Selected Stimuli pictures were randomized with respect to order of presentation. Except as already noted (sequentially identical for bar and arms with the Selected Stimuli) the B+A Stimuli were also randomized. The Single Stimuli orders were also randomized except that again in the B Order the bar was sequentially identical with the order for the Selected Stimuli and in the L Order the legs were sequentially identical with the Selected Stimuli. The same relationship held for the A and H Order treatments.

In addition to their assigned stimulus treatment, each group of subjects also received the Constant Stimuli. Thus each subject responded to 800 pictures. The Constant Stimuli were interspersed between the specially assigned stimuli. For example, a subject in the Correlated Stimulus Treatment Group was first given 90 trials using pictures drawn entirely from the Correlated Stimuli. Thereafter the presentations were alternated between the Constant Stimuli and the Correlated Stimuli until the latter were exhausted. The final 90 trials were completely made up of the Constant Stimuli.

Although the procedures were basically identical, it was convenient to use a different apparatus with the retarded subjects from that used with the normals.

For the latter the assigned stimuli and the Constant Stimuli were loaded into different projectors. These were both focused on the same screen and by means of a specially built shutter mechanism the alternating process was accomplished. These subjects simply looked at the pictures and indicated on prepared data sheets whether they believed the particular presentation was an X or a Y. They were given 20 seconds to make their decision and record their response. After the answer was written, the subjects were given the correct answer (i.e., knowledge of results) to those pictures which belonged to the stimulus groups to which they were assigned. They were not given any knowledge of results concerning the Constant Stimuli. For example, a subject assigned to the Correlated Stimulus Group was shown on his 91st trial the first picture of the Constant Stimulus Group. After his answer was recorded the experimenter immediately showed him for his 92nd trial the 91st picture of the Correlated Stimuli. Again he was given 20 seconds to respond. After response he was told whether the picture was actually an X or a Y, and the 93rd trial was begun using the 2nd picture from the Constant Stimuli, etc. These subjects were given to understand that they would be paid in accordance with the number of correct responses that they made to both the Correlated and Constant Stimuli. They were never aware, of course, that two different groups of stimuli were involved. They only knew that knowledge of results did not always follow a response. Answers were not given to the Constant Stimuli for the reason that there were no answers to give. To have assigned answers to these stimuli would have disturbed the probability functions operating in the Correlated Stimuli. The purpose of the Constant Stimuli was to sample the probability functions without modifying these parameters by knowledge of results.

The retarded subjects faced a translucent screen. The pictures were projected from behind the screen, and the subject responded by pressing one of the two available pushbutton switches (X or Y). If the correct answer was given, a green light came on and a piece of candy was delivered down the chute. If the answer was incorrect, a red light came on. This procedure was followed because previous research had shown that these subjects required immediate and non-symbolic reinforcement and because they had some difficulty in associating a data sheet with the picture. They could see no causal relationship between marks on a piece of paper and reinforcement. To them the occurrence or absence of reinforcement as a consequence of writing a check mark was an arbitrary act on the part of the tester. Pushing a button, however, was a more direct and instrumental act. It never seemed to occur to them that the experimenter was involved in the process. It would almost appear that to the retarded machines are orderly and rational while humans are simply capricious and willful. They did not display the anger toward the machine that they would have shown toward the experimenter if he had simply handed them the candy.

RESULTS

The results of the analyses of the experimental data are shown in Figures 6 through 33. Two methods were available for estimating the strength of the individual cues. One was based upon the results of the four orders obtained from the Single Stimulus Group and the other upon the results from the Constant Stimuli. The Single Stimulus method worked in the following way. If on the n th trial 57% of the B Order Group responded to their picture as an X, 30% of the L Order Group

Figure 6. Correlations obtained for High School Group on Selected Stimuli problem based on Single Stimuli data for the three synthesis formulas.

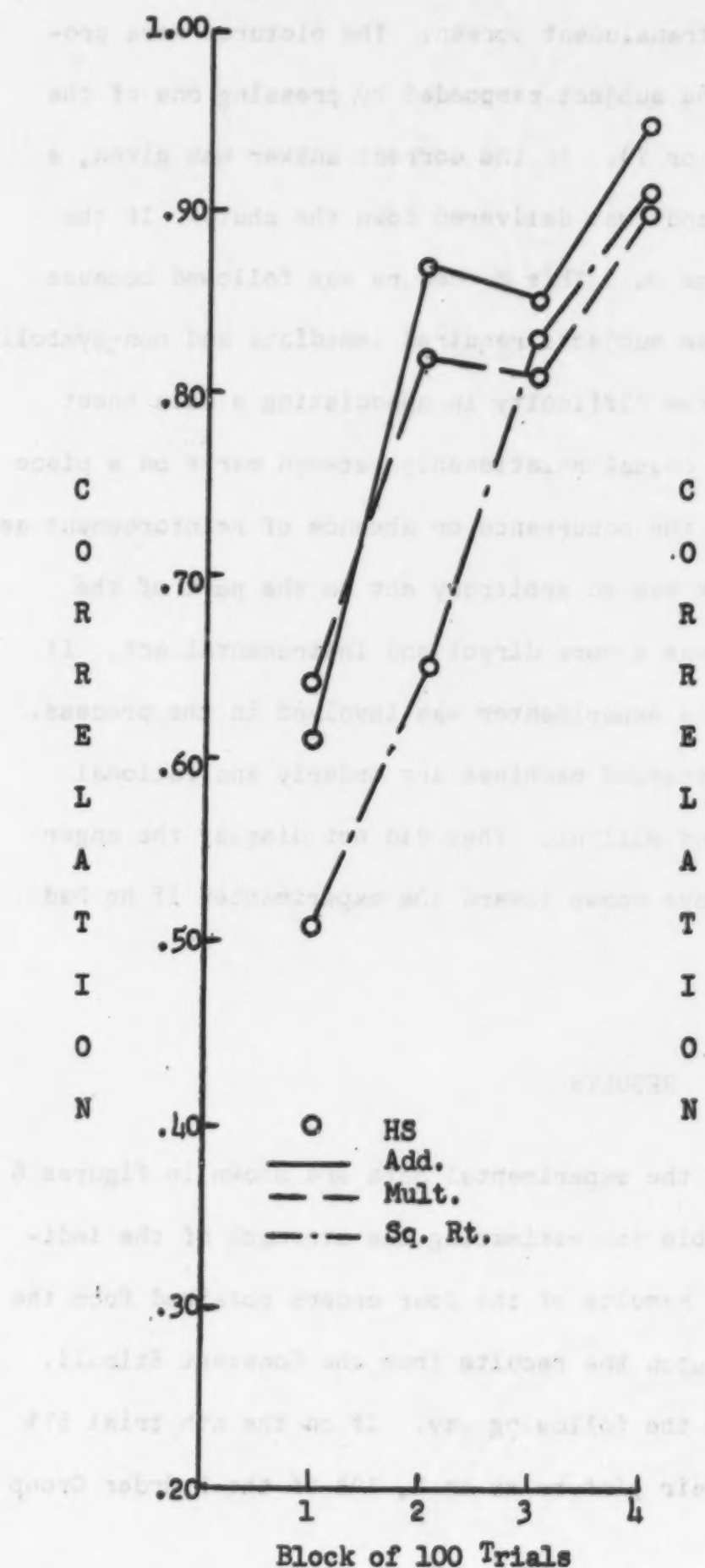


Figure 7. Correlations obtained for Grade School Group on Selected Stimuli problem based on Single Stimuli data for the three synthesis formulas.

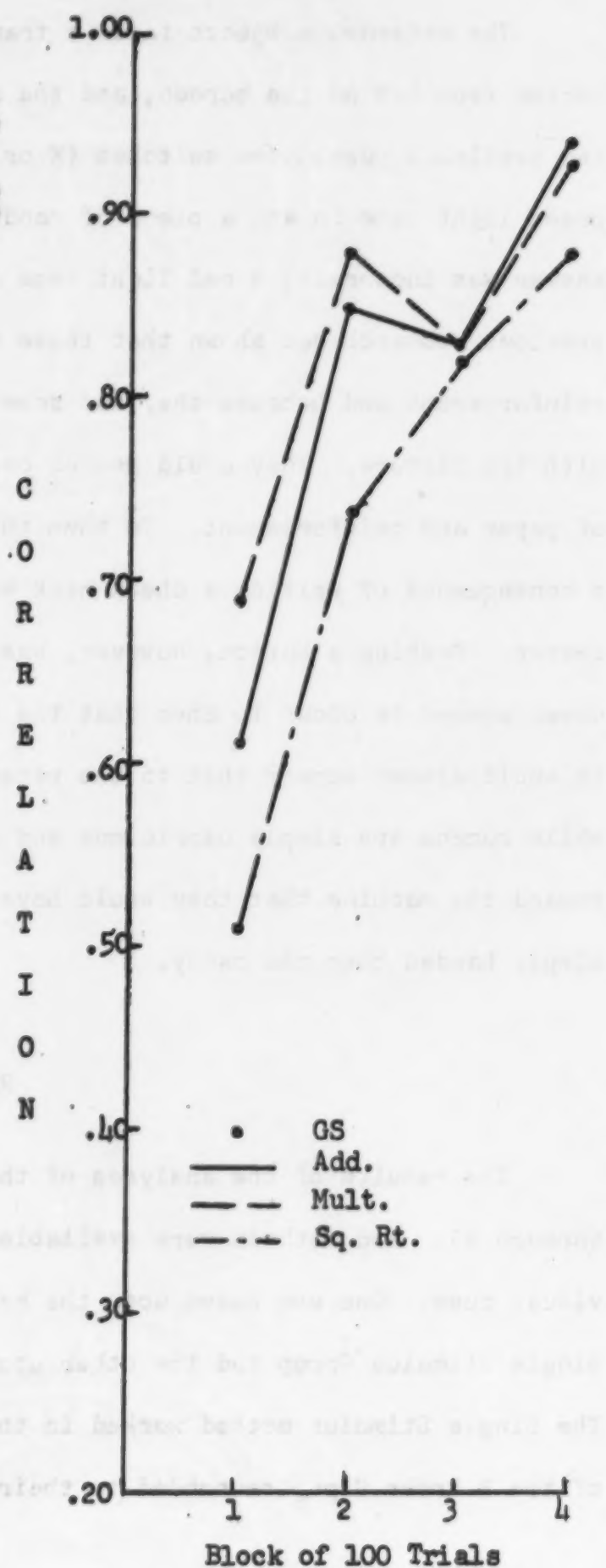


Figure 8. Correlations obtained for Retarded Group on Selected Stimuli problem based on Single Stimuli data for the three synthesis formulas.

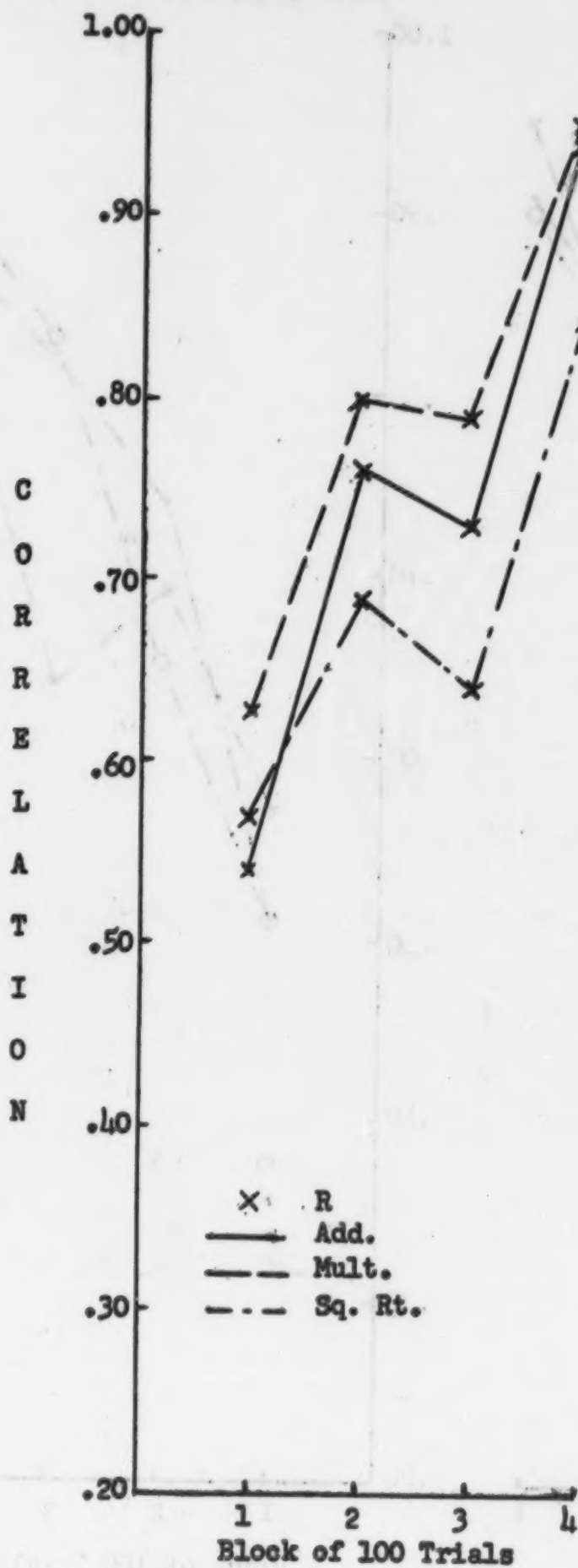


Figure 9. Correlations obtained for additive solution on Selected Stimuli problem based on Single Stimuli data for the three groups of subjects.

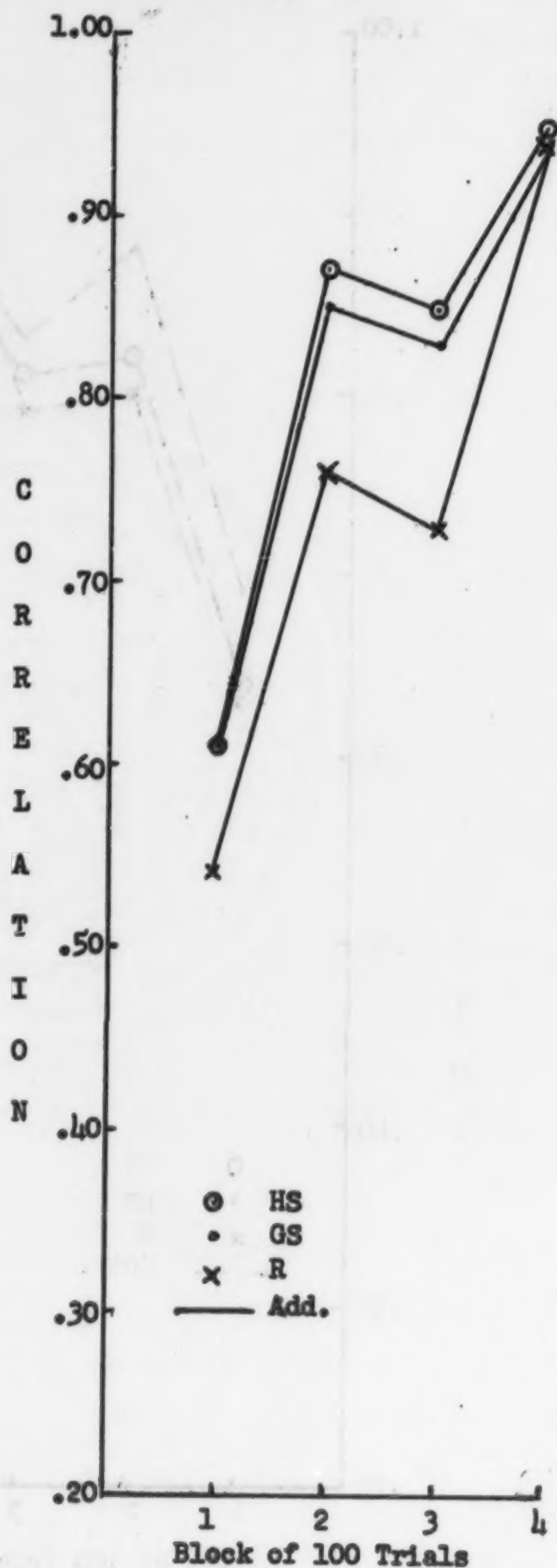


Figure 10. Correlations obtained for multiplicative solution on Selected Stimuli problem based on Single Stimuli data for the three groups of subjects.

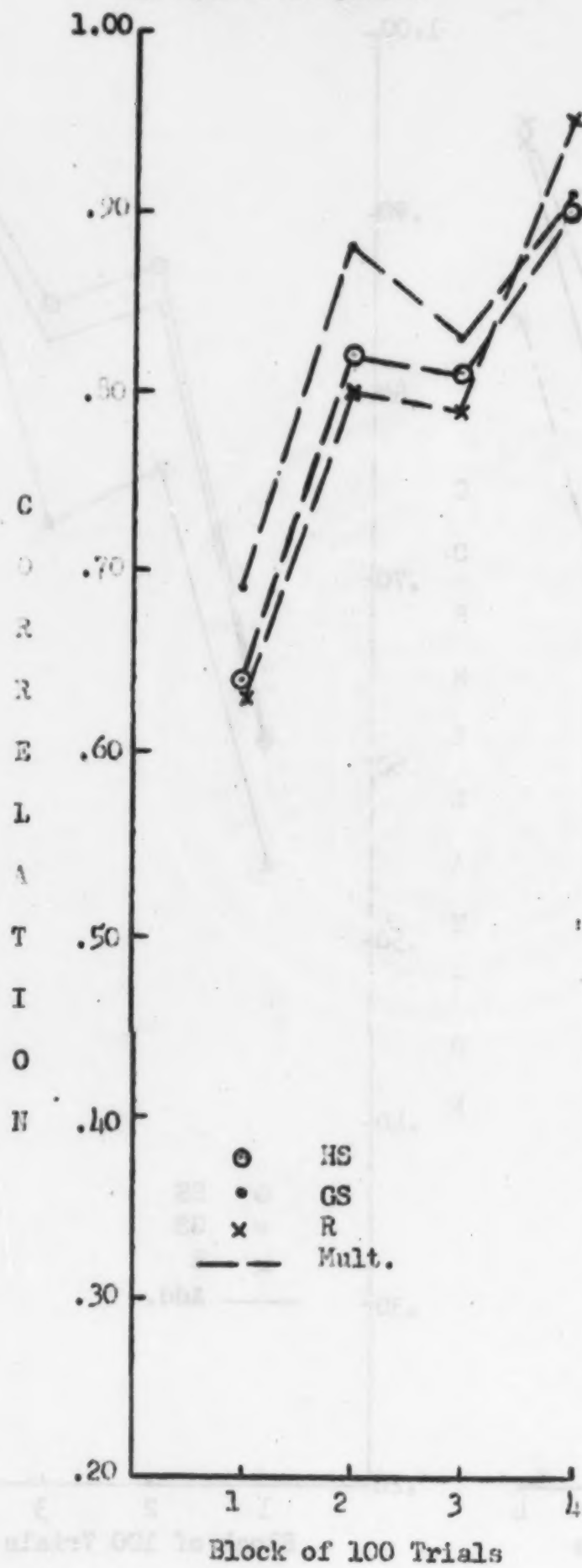


Figure 11. Correlations obtained for square root solution on Selected Stimuli problem based on Single Stimuli data for the three groups of subjects.

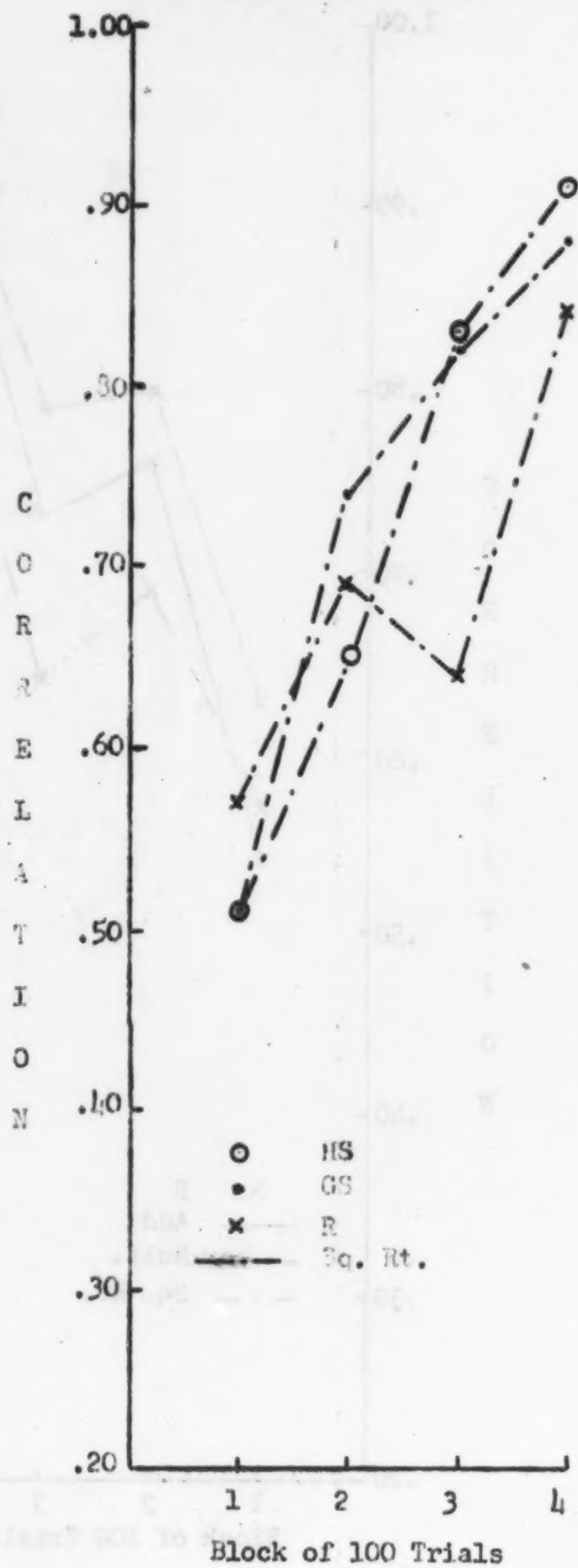


Figure 12. Correlations obtained for High School Group on Selected Stimuli problem based on Constant Stimuli data for the three synthesis formulas.

Figure 13. Correlations obtained for Grade School Group on Selected Stimuli problem based on Constant Stimuli data for the three synthesis formulas.

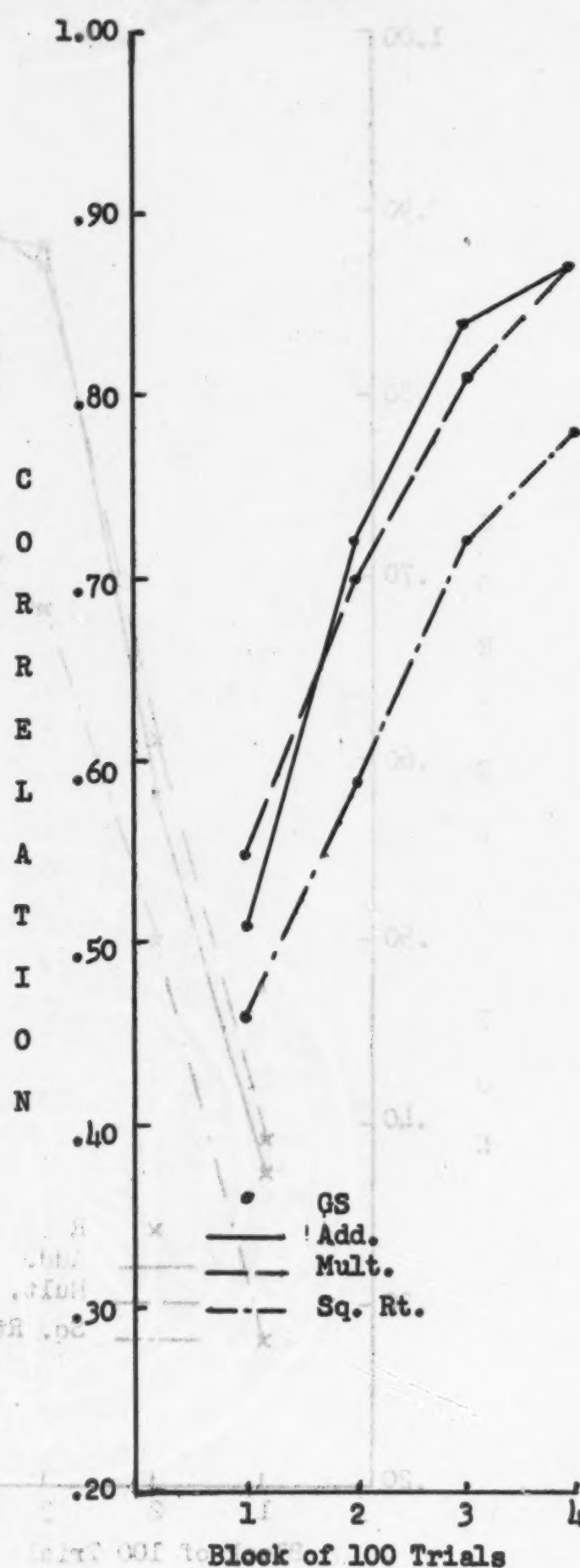
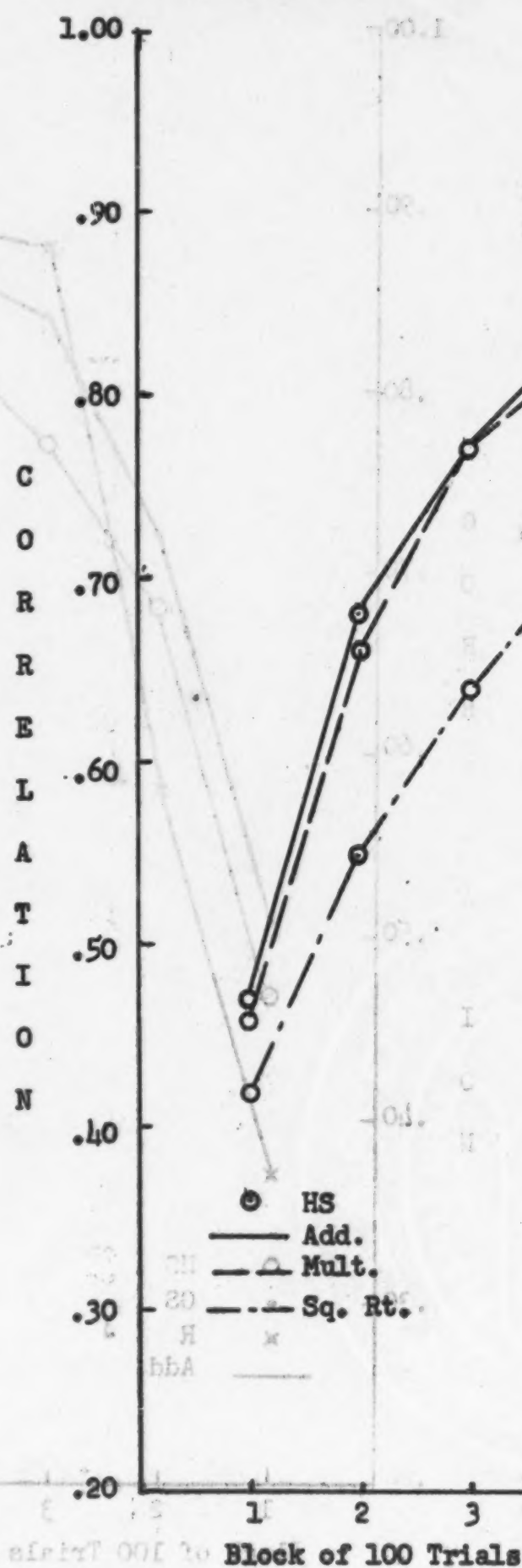


Figure 14. Correlations obtained for Retarded Group on Selected Stimuli problem based on Constant Stimuli data for the three synthesis formulas.

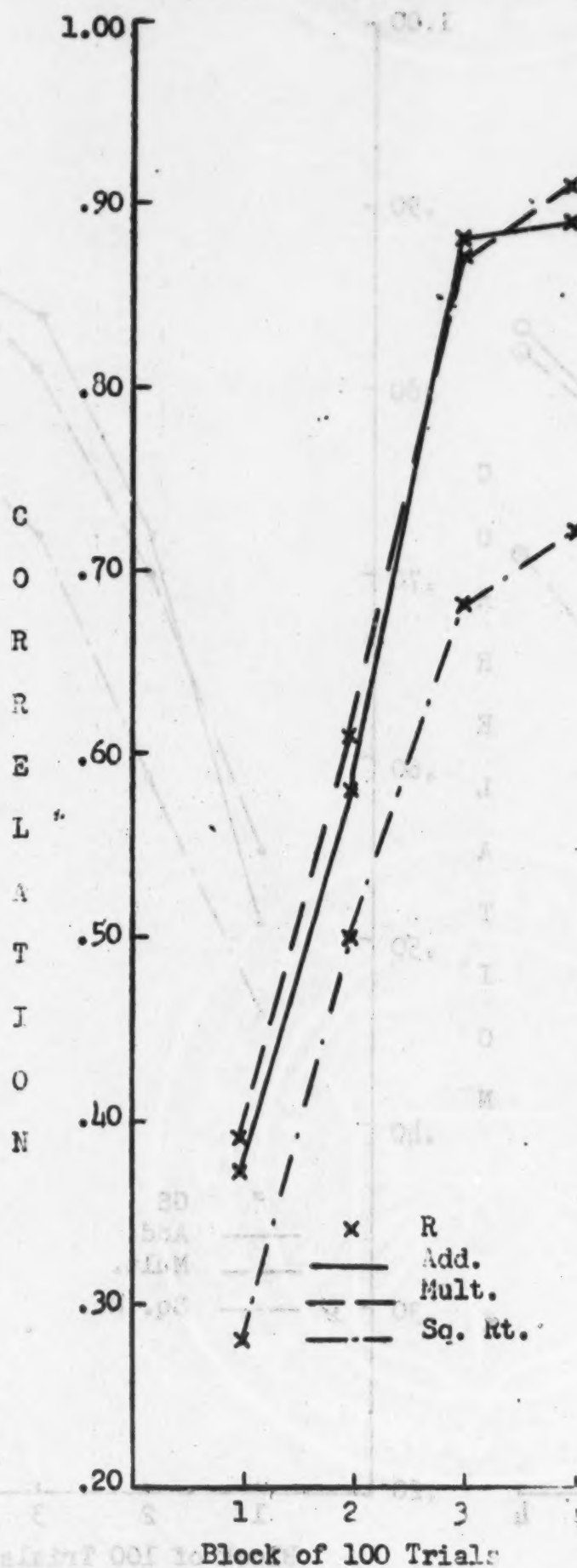


Figure 15. Correlations obtained for additive solution on Selected Stimuli problem based on Constant Stimuli data for the three groups of subjects.

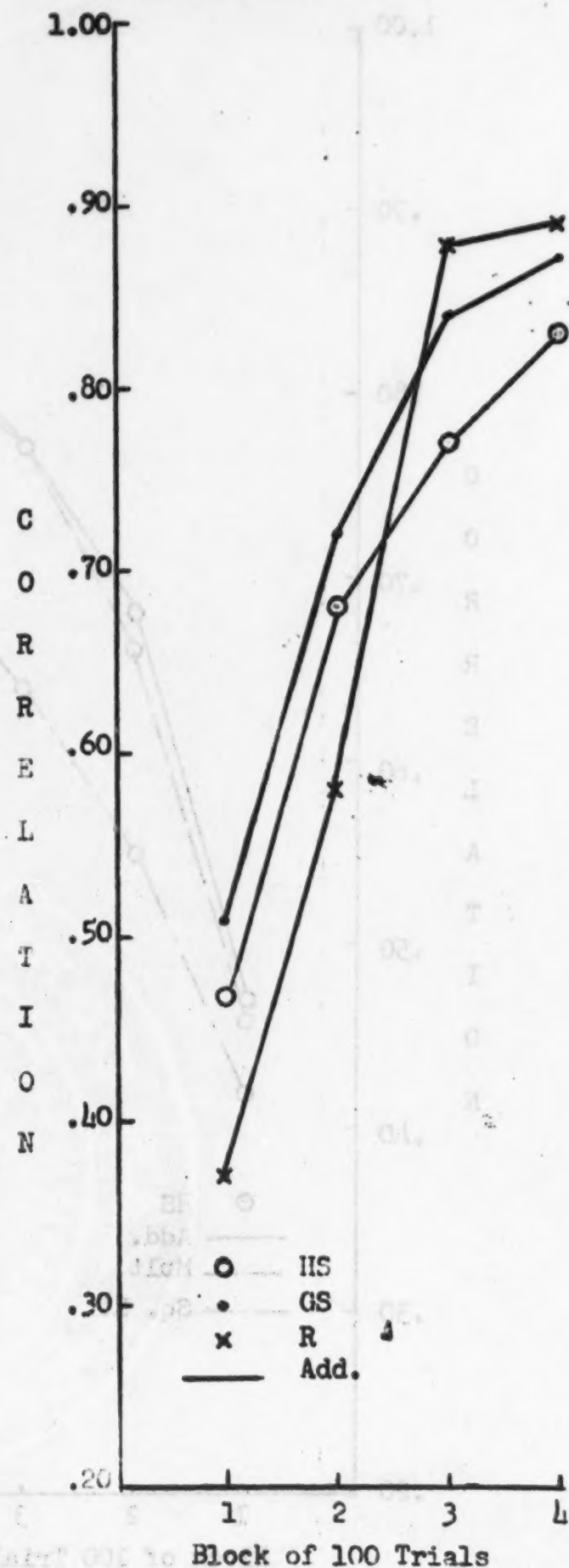


Figure 16. Correlations obtained for multiplicative solution on Selected Stimuli problem based on Constant Stimuli data for the three groups of subjects.

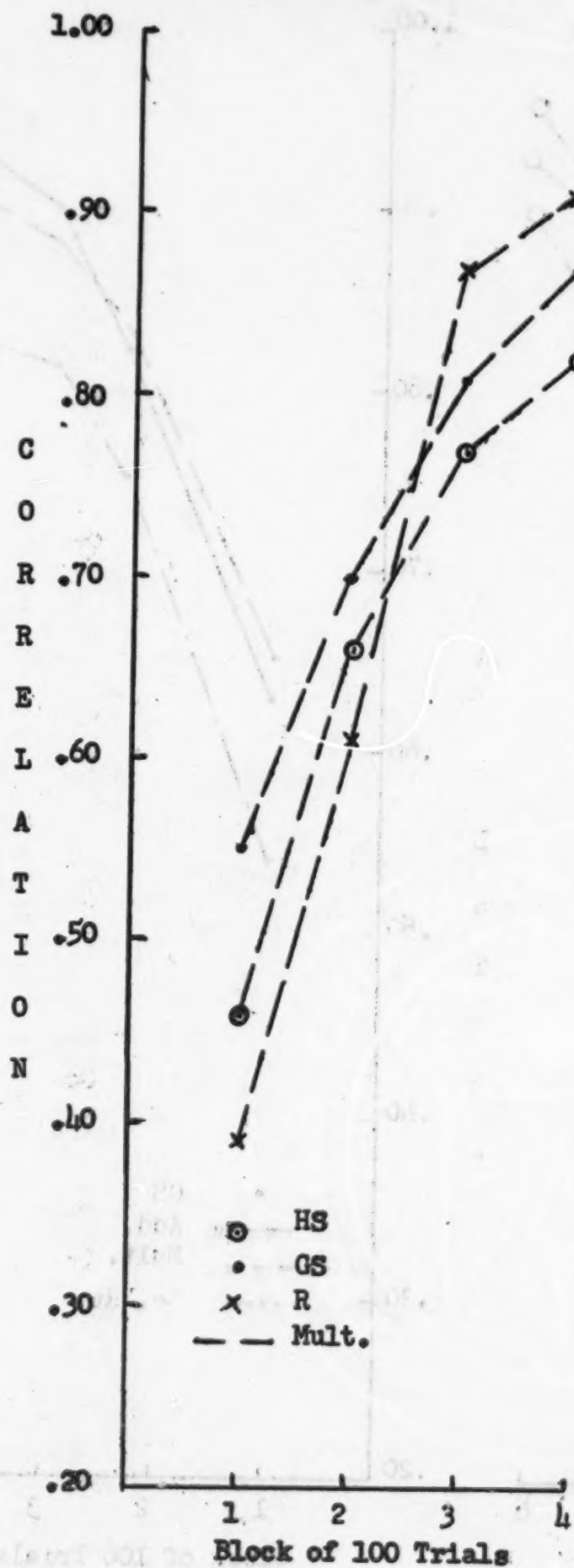


Figure 17. Correlations obtained for square root solution on Selected Stimuli problem based on Constant Stimuli data for the three groups of subjects.

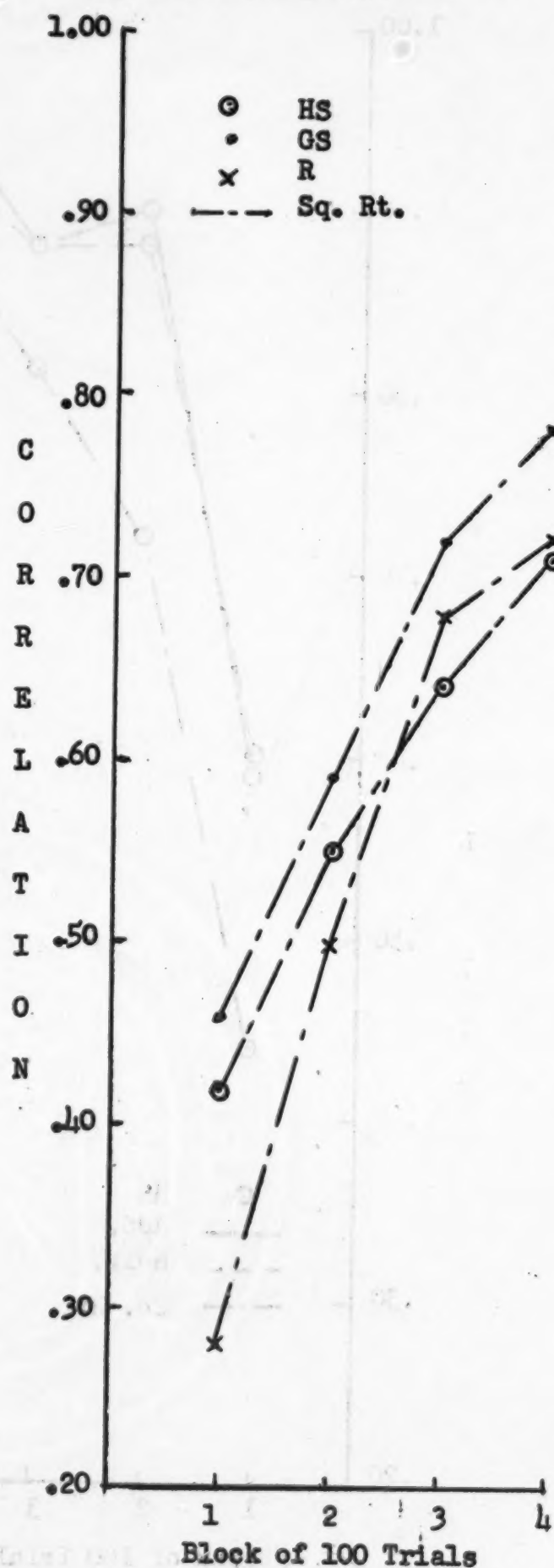


Figure 18. Correlations obtained for High School Group on Correlated Stimuli problem based on Constant Stimuli data for the three synthesis formulas.

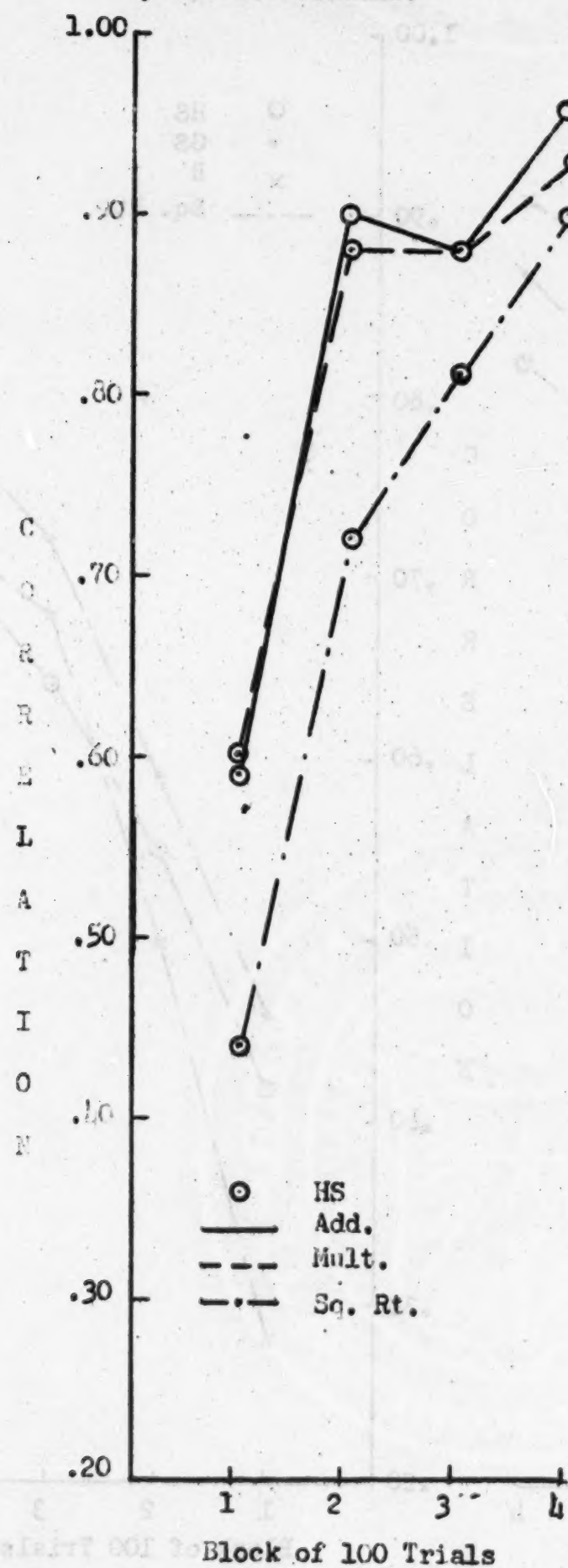


Figure 19. Correlations obtained for Grade School Group on Correlated Stimuli problem based on Constant Stimuli data for the three synthesis formulas.

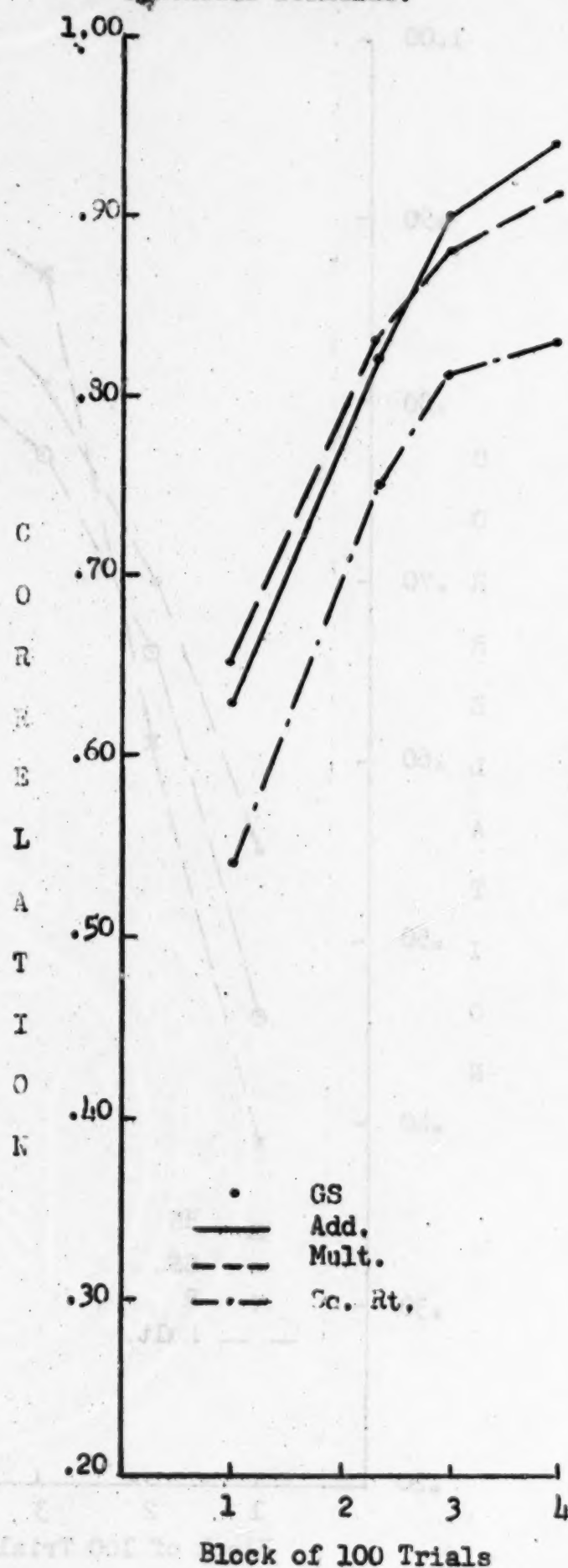


Figure 20. Correlations obtained for additive solution on Correlated Stimuli problem based on Constant Stimuli data for the two groups of subjects.

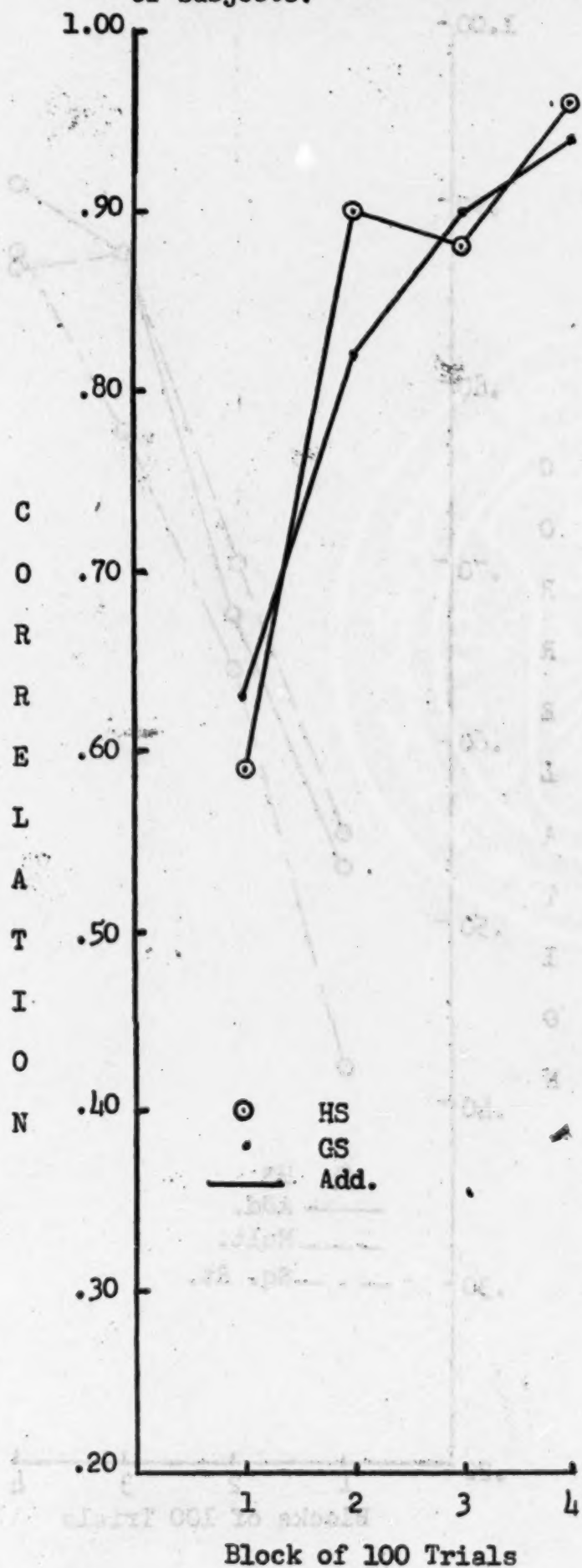


Figure 21. Correlations obtained for multiplicative solution on Correlated Stimuli problem based on Constant Stimuli data for the two groups of subjects.

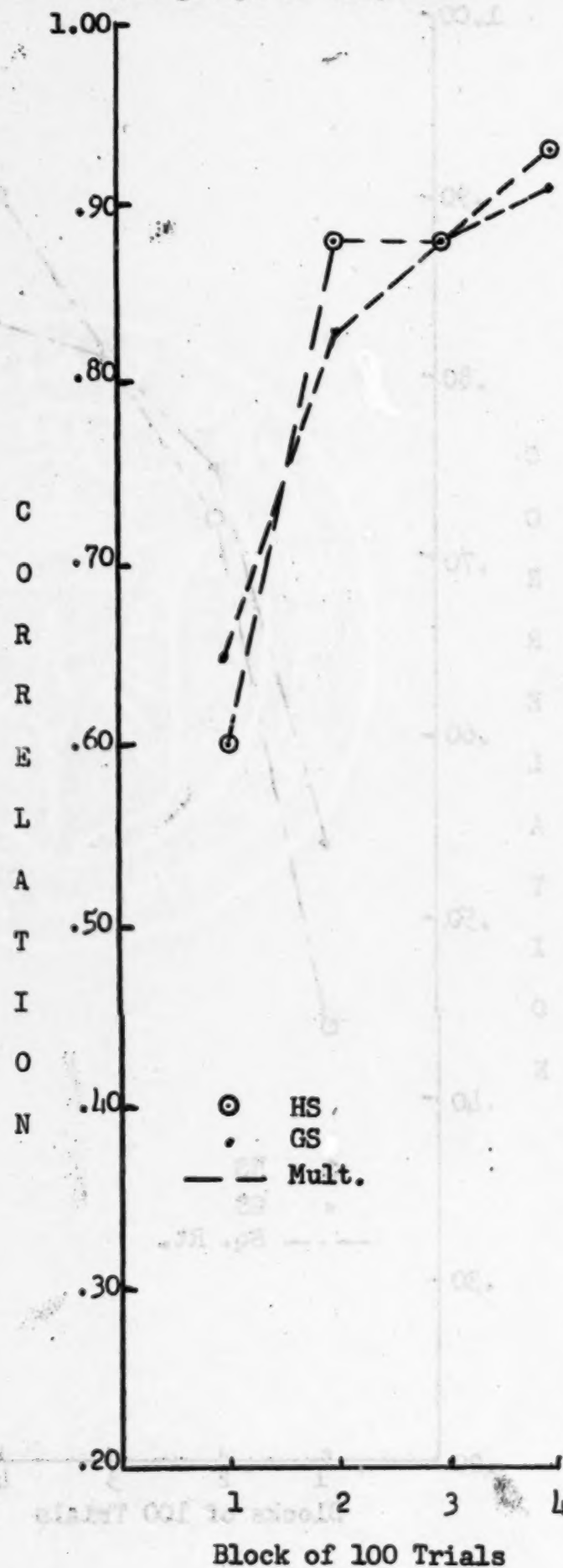


Figure 22. Correlations obtained for square root solution on Correlated Stimuli problem based on Constant Stimuli data for the two groups of subjects.

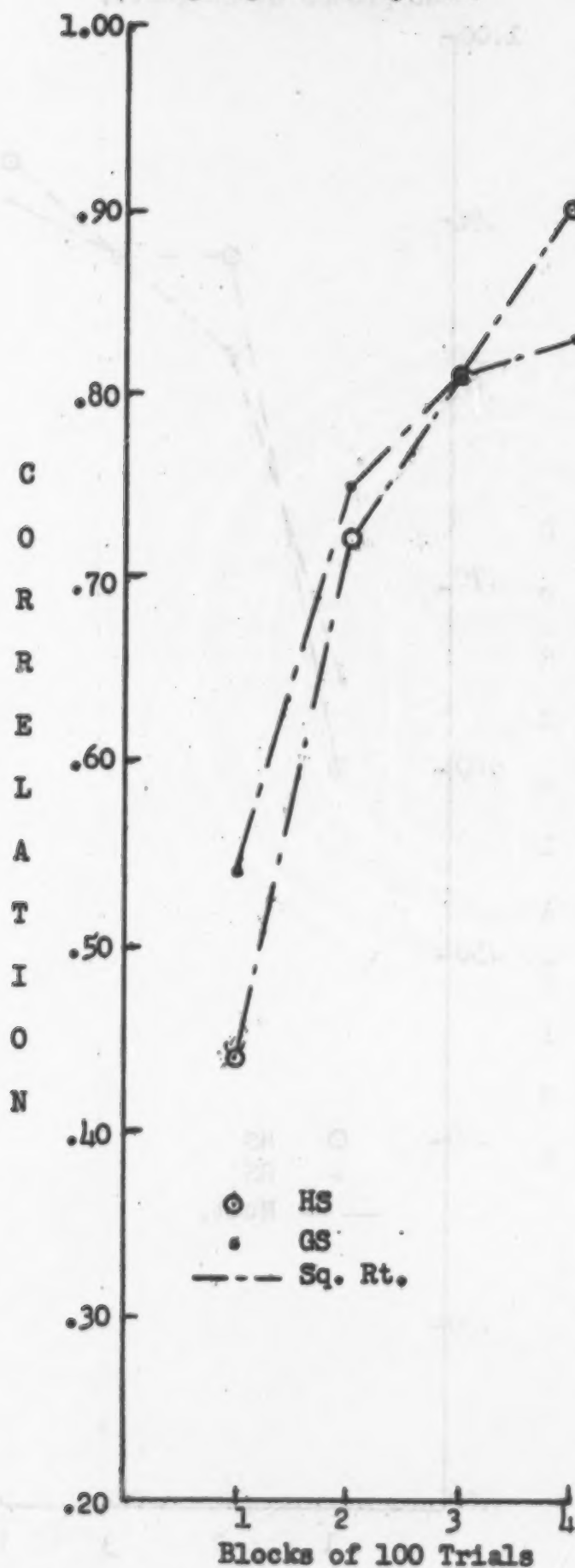


Figure 23. Correlations obtained for High School Group on Random Stimuli problem based on Constant Stimuli data for the three synthesis formulas.

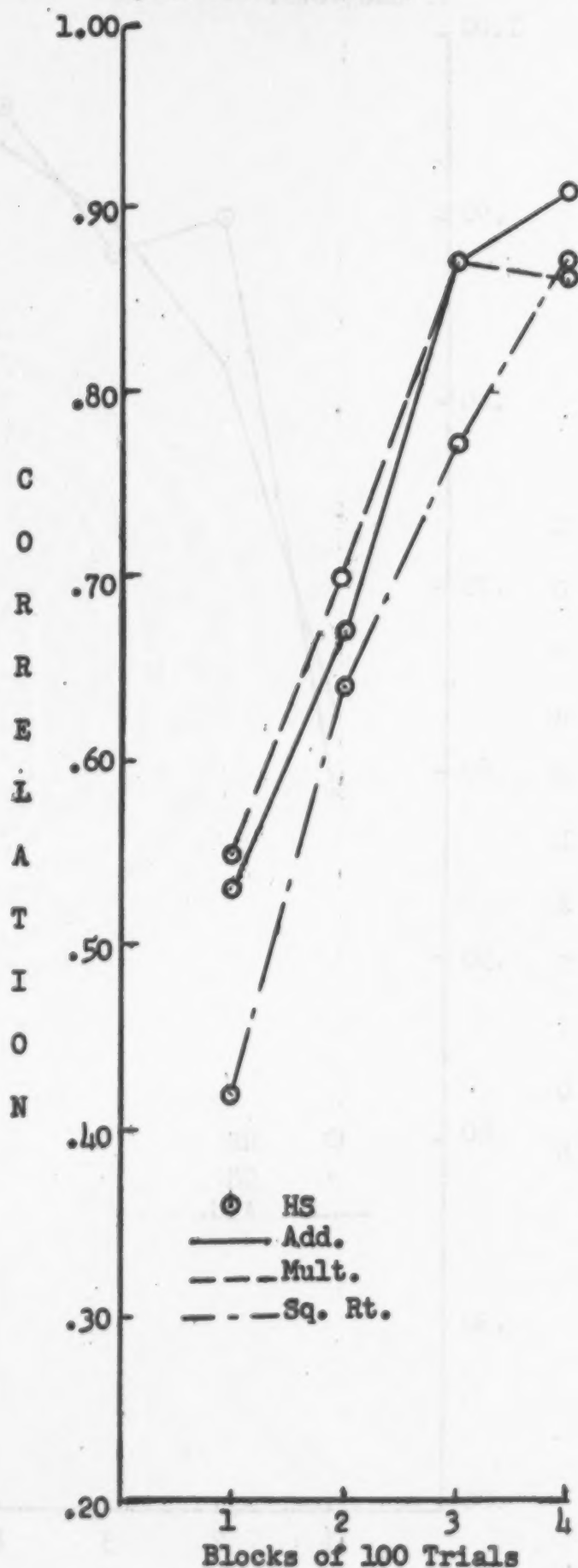


Figure 24. Correlations obtained for Grade School Group on Random Stimuli problem based on Constant Stimuli data for the three synthesis formulas.

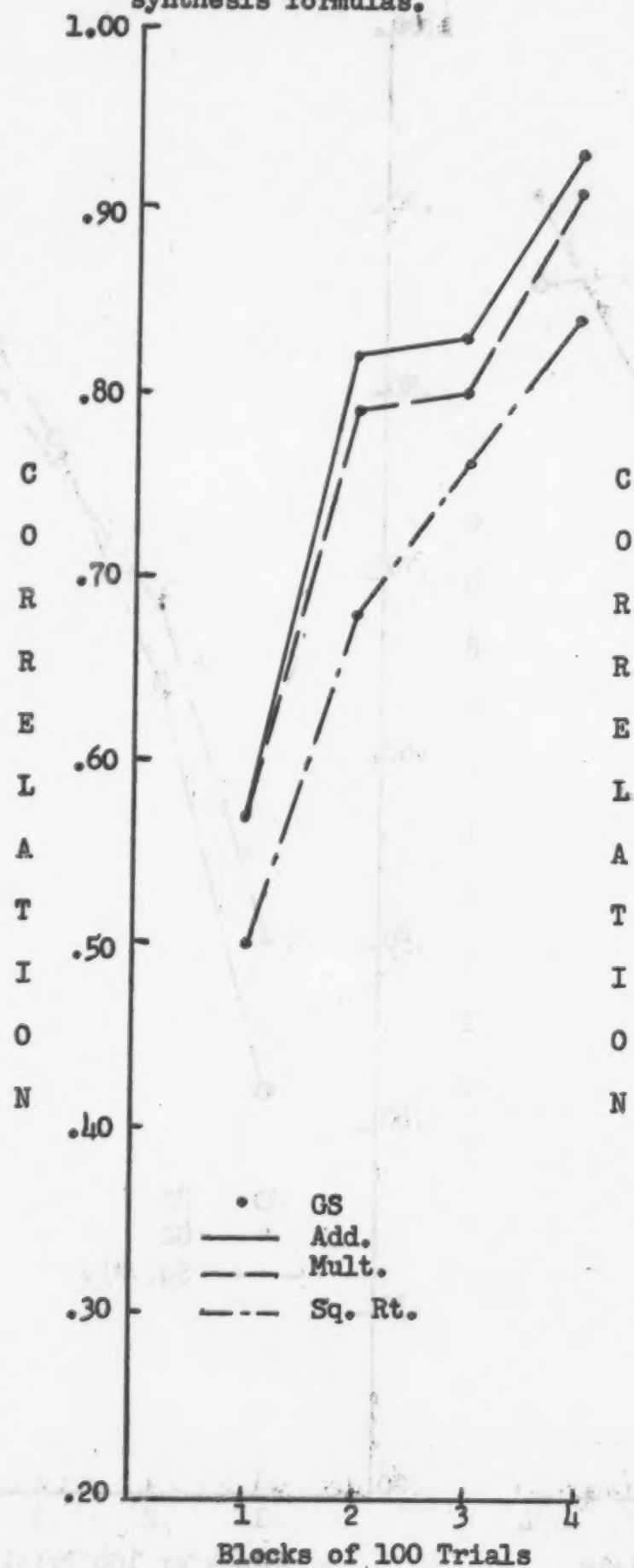


Figure 25. Correlations obtained for additive solution on Random Stimuli problem based on Constant Stimuli data for the two groups of subjects.

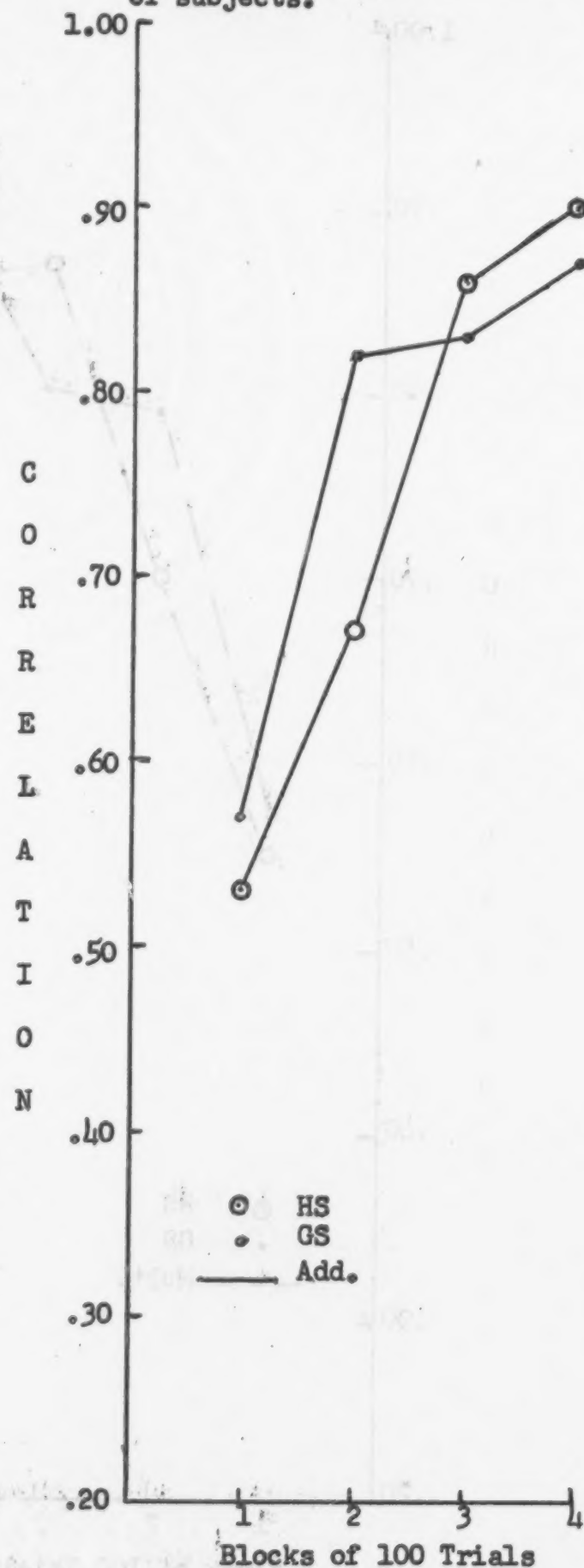


Figure 26. Correlations obtained for multiplicative solution on Random Stimuli problem based on Constant Stimuli data for the two groups of subjects.

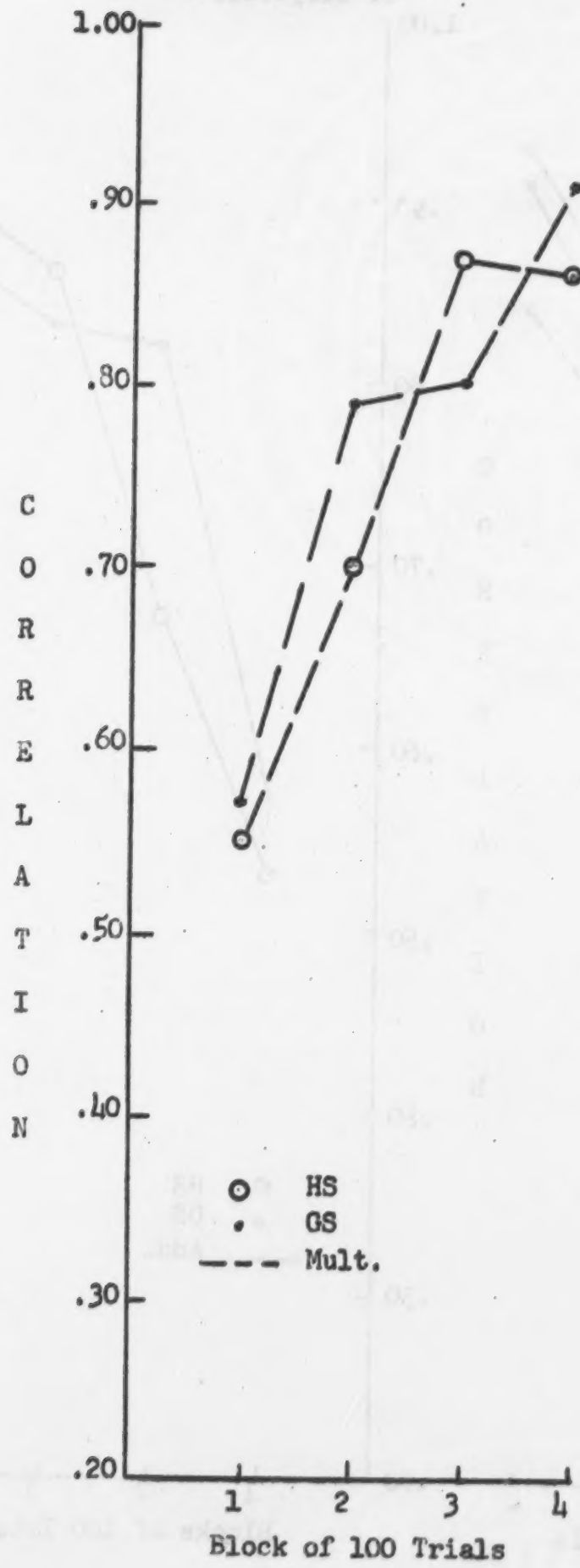


Figure 27. Correlations obtained for square root solution on Random Stimuli problem based on Constant Stimuli data for the two groups of subjects.

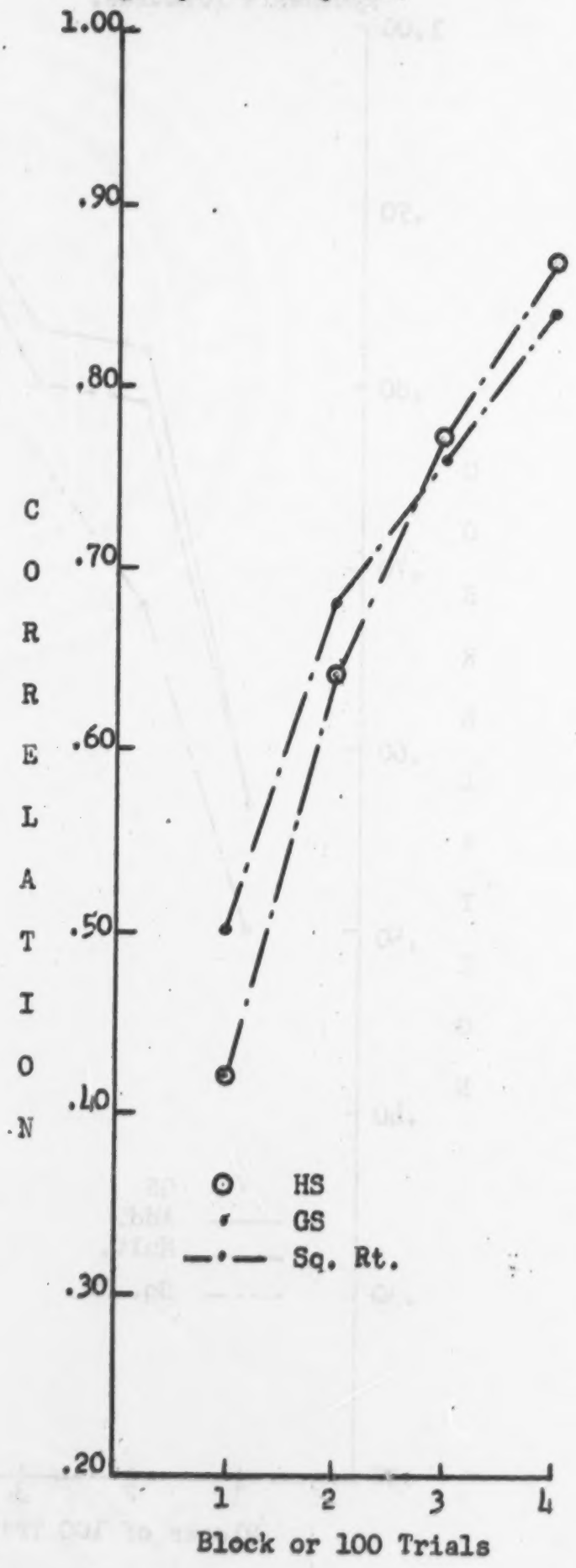


Figure 28. Correlations obtained for High School Group on B + A Stimuli problem based on Single Stimuli data for the three synthesis formulas.

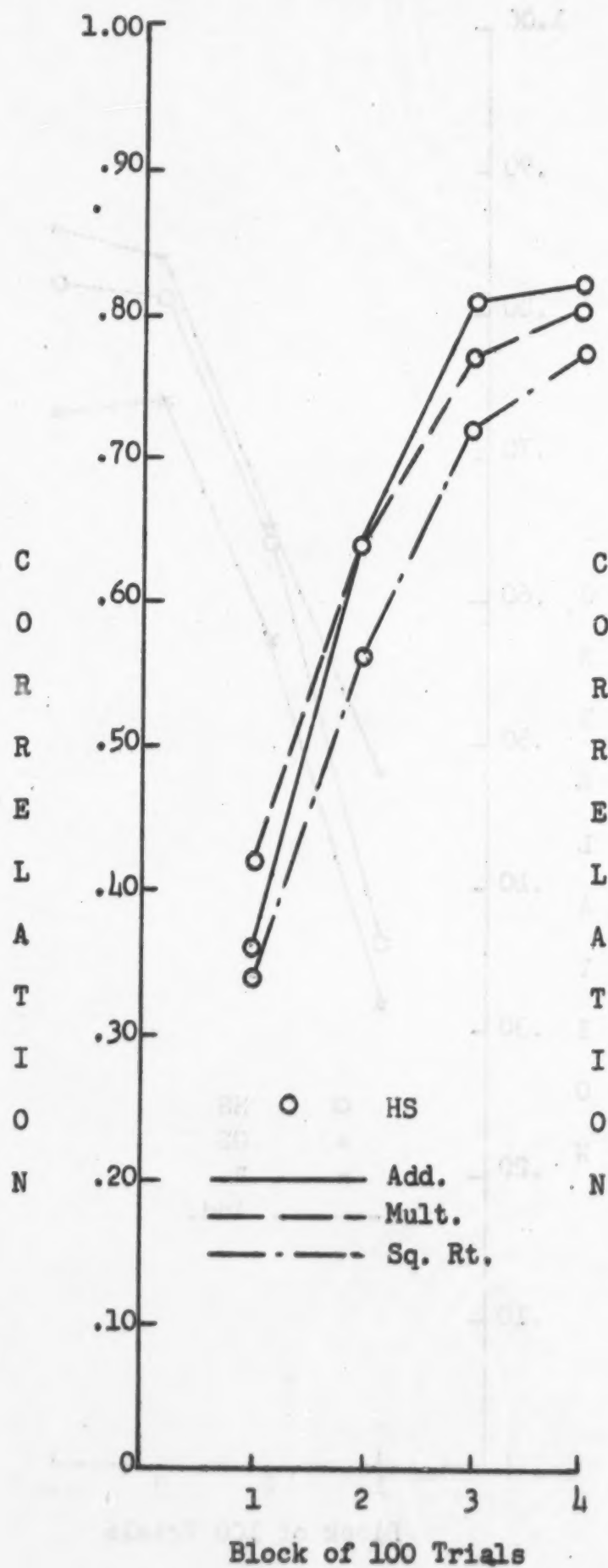


Figure 29. Correlations obtained for Grade School Group on B + A Stimuli problem based on Single Stimuli data for the three synthesis formulas.

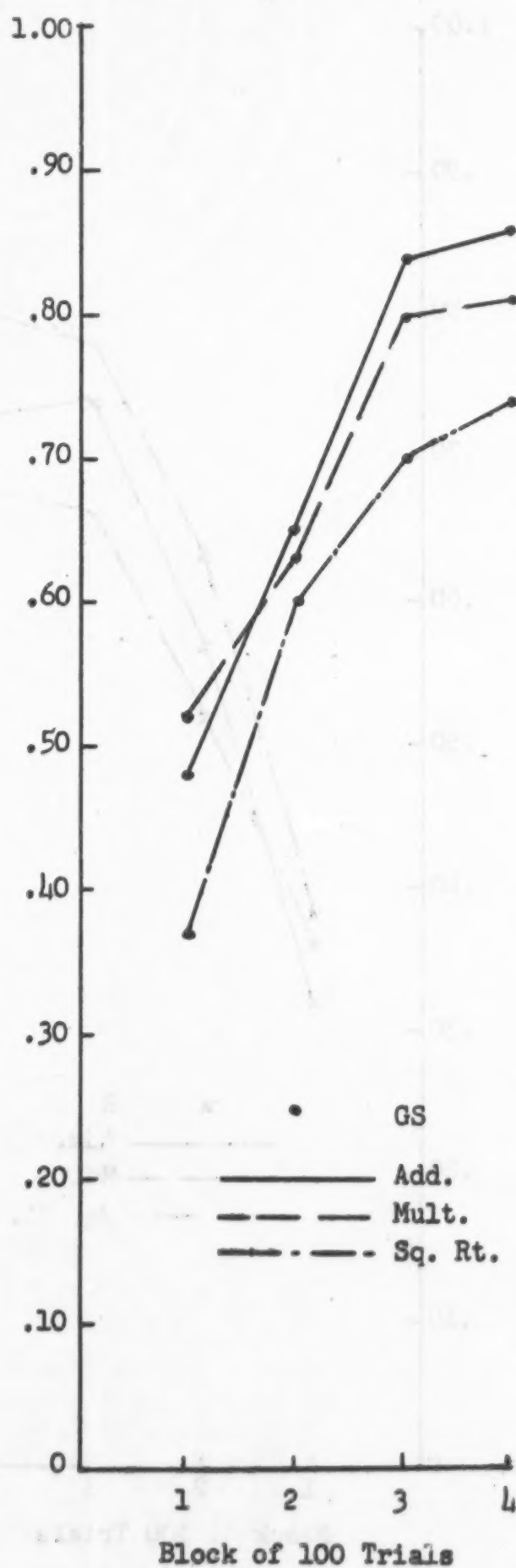


Figure 30. Correlations obtained for Retarded Group on B + A Stimuli problem based on Single Stimuli data for the three synthesis formulas.

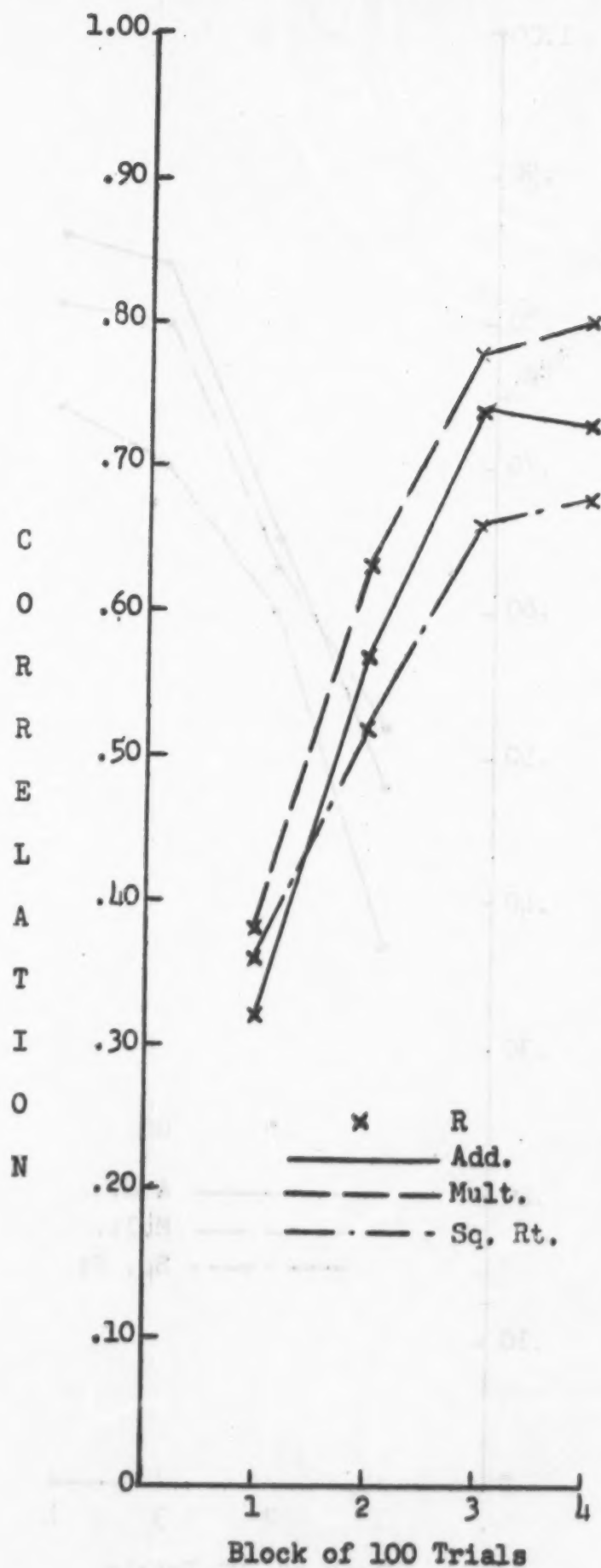


Figure 31. Correlations obtained for additive solution on B + A Stimuli problem based on Single Stimuli data for the three groups of subjects.

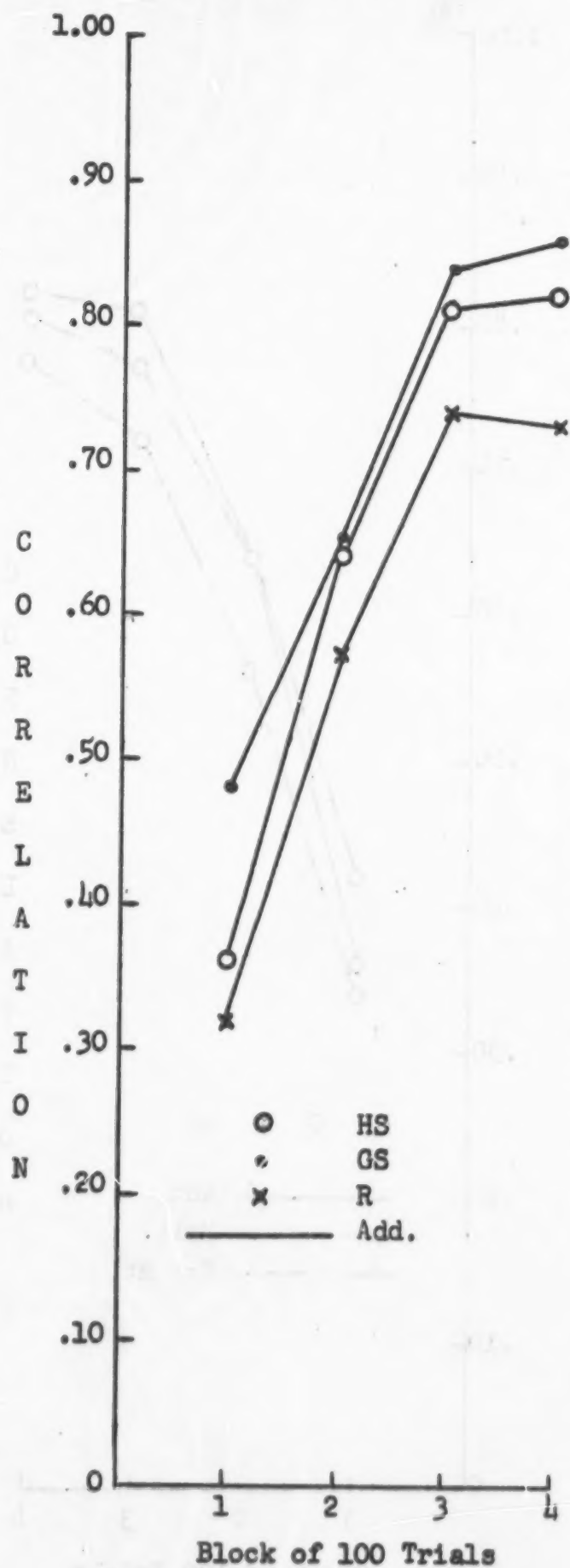


Figure 32. Correlations obtained for multiplicative solution on B + A Stimuli problem based on Single Stimuli data for the three groups of subjects.

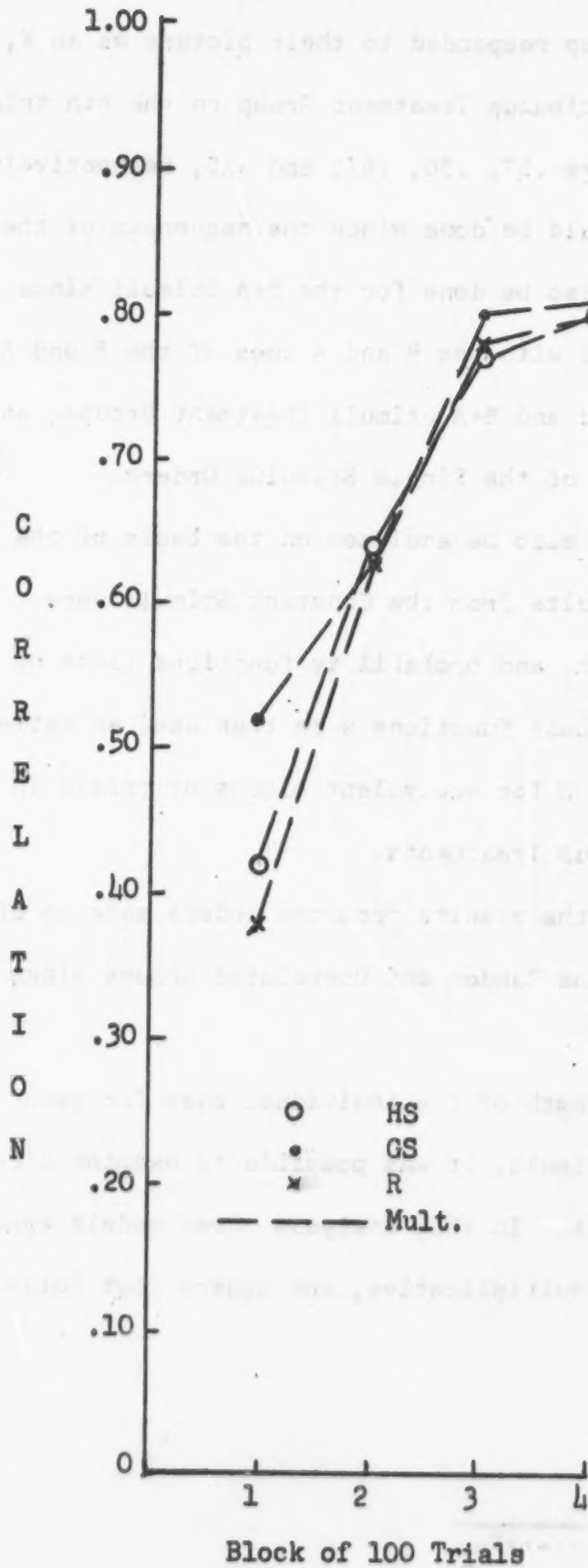
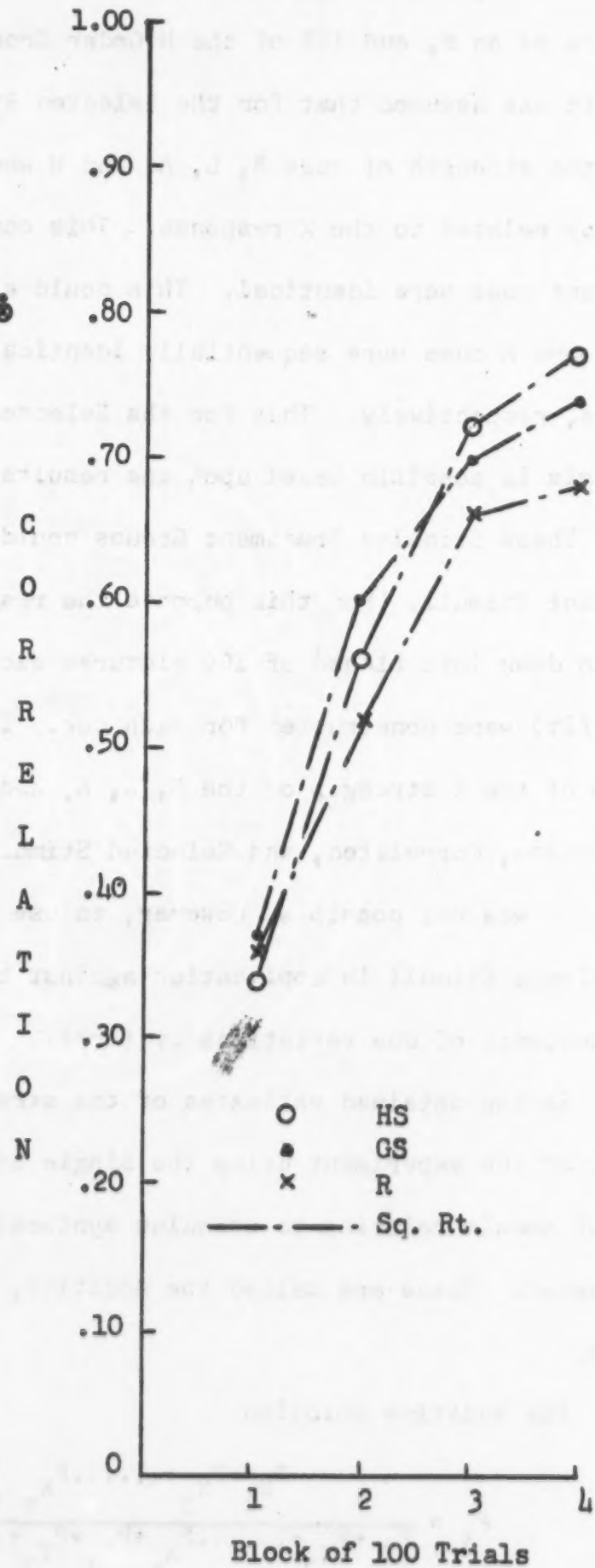


Figure 33. Correlations obtained for square root solution on B + A Stimuli problem based on Single Stimuli data for the three groups of subjects.



responded to their picture as an X, 67% of the A Order Group responded to their picture as an X, and 15% of the H Order Group responded to their picture as an X, then it was assumed that for the Selected Stimulus Treatment Group on the nth trial that the strength of cues B, L, A, and H were .57, .30, .67, and .15, respectively, as they related to the X response. This could be done since the sequences of the relevant cues were identical. This could also be done for the B+A Stimuli since the B and A cues were sequentially identical with the B and A cues of the B and A Orders, respectively. Thus for the Selected and B+A Stimuli Treatment Groups, an analysis is possible based upon the results of the Single Stimulus Orders.

These Stimulus Treatment Groups could also be analyzed on the basis of the Constant Stimuli. For this purpose the results from the Constant Stimuli were broken down into blocks of 100 pictures each, and probability functions (line of best fit) were constructed for each cue. These functions were then used as estimates of the X strength of the B, L, A, and H for equivalent blocks of trials in the Random, Correlated, and Selected Stimulus Treatments.

It was not possible, however, to use the results from the orders made up of the Single Stimuli in application against the Random and Correlated Groups since the sequence of cue variations differed.

Having obtained estimates of the strength of the individual cues for each trial of the experiment using the Single Stimuli, it was possible to examine different models relating to stimulus synthesis. In this analysis three models were evaluated. These are called the Additive, Multiplicative, and Square Root Solutions.

The Additive Solution

$$P_X = \frac{P_{X_1} + P_{X_2} + \dots + P_{X_n}}{P_{X_1} + P_{X_2} + \dots + P_{X_n} + P_{Y_1} + P_{Y_2} + \dots + P_{Y_n}}$$

simply states that the stimulus synthesis is a linear process. Supposing that all four cues were used on the nth trial described above, then for the Single Stimulus Method

$$P_X = \frac{.57 + .30 + .67 + .15}{.57 + .30 + .67 + .15 + .43 + .70 + .33 + .85} = .42.$$

This means that the probability of a person responding on the nth trial as an X is .42. It also follows that 42% of the Selected Stimulus Treatment Group will respond to the picture as an X. Thus we can compare the predicted result against the actual result. The most straight-forward method of making the comparison (although it is not the most sensitive) is to use the correlation between the predicted and actual outcome for each trial. Since, however, it is desirable to witness the efficacy of the model through time, it is convenient to break down the experiment into blocks of 100 trials and produce correlations based upon each block. This has been done for Figures 6 through 33.

The second model is called the Multiplicative Solution and has the formula

$$P_X = \frac{P_{X_1} P_{X_2} \dots P_{X_n}}{(P_{X_1} P_{X_2} \dots P_{X_n}) + (P_{Y_1} P_{Y_2} \dots P_{Y_n})}$$

In essence this model states that the subject emphasizes the extreme cue values at the expense of the more central ones. In the example used above

$$P_X = \frac{.57 .30 .67 .15}{(.57 .30 .67 .15) + (.43 .70 .33 .85)} = .17.$$

This value is lower than that derived by the Additive Formula. This is primarily due to the low value for H (.15).

In the final model the assumption is made that the subject emphasizes the central values and underestimates the extreme cues.

$$P_X = \frac{\sqrt{P_{X_1} P_{X_2} \dots P_{X_n}}}{(\sqrt{P_{X_1} P_{X_2} \dots P_{X_n}}) + (\sqrt{P_{Y_1} P_{Y_2} \dots P_{Y_n}})}$$

or

$$P_X = \frac{\sqrt{.50 \ .30 \ .67 \ .15}}{(\sqrt{.50 \ .30 \ .67 \ .15}) + (\sqrt{.43 \ .70 \ .33 \ .85})} = .46.$$

This value is greater than that shown for the Additive Solution.

Since there were four stimulus cues which were relevant to the solution of the problems, it follows that there were 15 possible solutions to each problem based upon the available combinations. These are:

1. Solution based upon B alone.
2. Solution based upon L alone.
3. Solution based upon A alone.
4. Solution based upon H alone.
5. Solution based upon the combination of B and L.
6. Solution based upon the combination of B and A.
7. Solution based upon the combination of B and H.
8. Solution based upon the combination of L and A.
9. Solution based upon the combination of L and H.
10. Solution based upon the combination of A and H.
11. Solution based upon the combination of B, L, and A.
12. Solution based upon the combination of B, L, and H.
13. Solution based upon the combination of B, A, and H.
14. Solution based upon the combination of L, A, and H.
15. Solution based upon the combination of B, L, A, and H.

Taking into account that examination was made of three stimulus integration solutions (Additive, Multiplicative, and Square Root) which could be applied against each of the combinational stimulus solutions (5 through 15 above) and also considering that a separate solution was obtained for each block of 100 trials, then it can be seen that for each stimulus treatment condition (Random, Correlated, Selected, and B+A) there were 148 $([(11 \times 3) + 4] \times 4)$ correlations to be run. As it was impossible to produce Single Stimulus solutions for the Random and Correlated Treatments and since the B+A Treatment was not analyzed using the Constant Stimuli, the total analysis amounted to 740 correlations. This is shown in Table 3.

Table 3

Methods of Analysis of Each Stimulus Combination for Each Synthesis Solution for Each Block of 100 Trials

Stimulus Treatment	Criteria	
	Single Stimuli	Constant Stimuli
Random		X
Correlated		X
Selected	X	X
B+A	X	

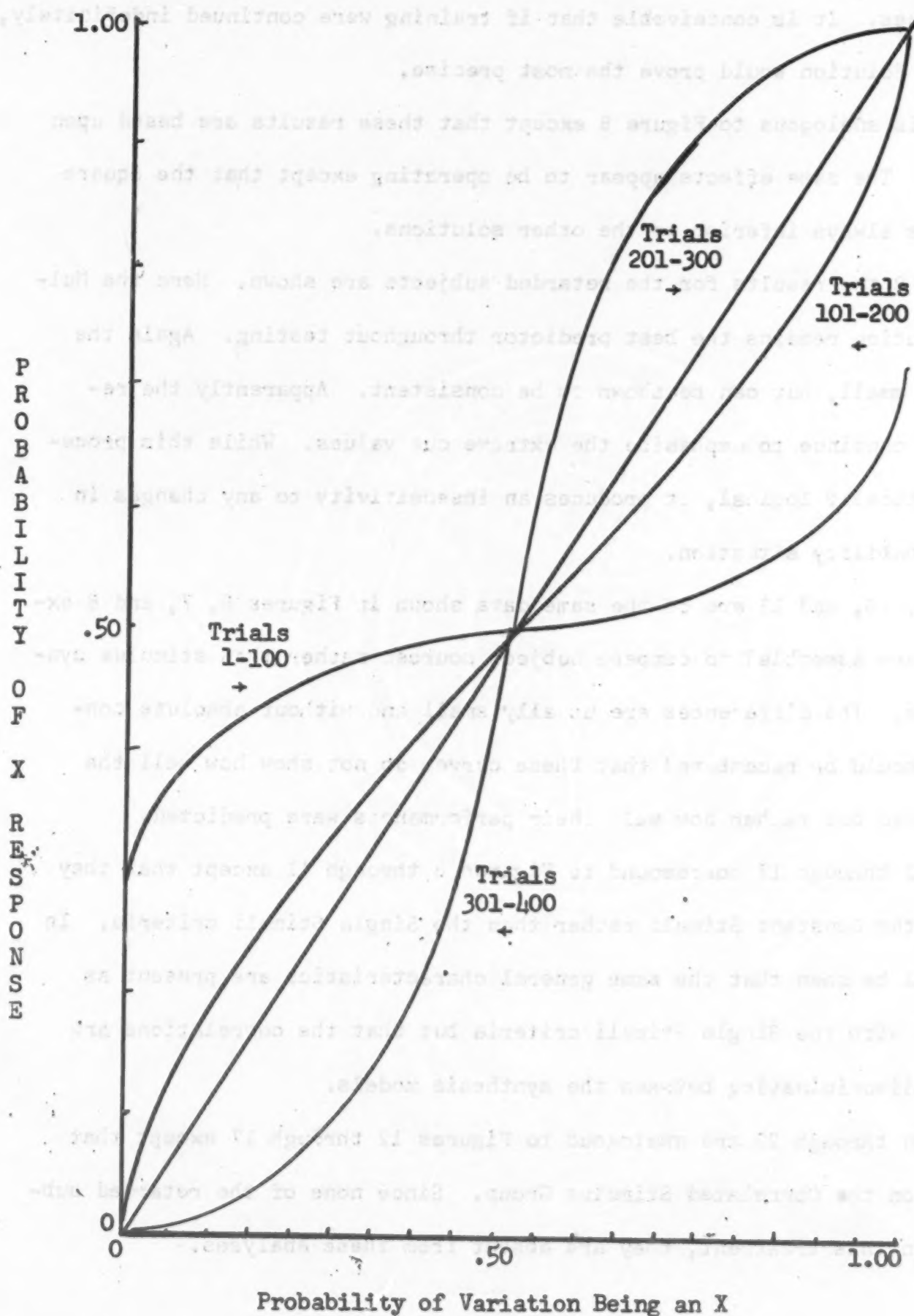
The combination of all stimuli (B, L, A, and H) produced the highest correlations of any combinations examined and only these are shown in Figures 6 through 33.

The analysis based upon the Single Stimuli had the advantage of being specific to each trial and thus reflected the progressive change in the strength

of the individual cues as the experiment evolved. This was not possible based upon the Constant Stimuli because the functions could be established only through a long series of trials. Since, however, it was desirable to reflect this continuous change for the analysis using the Constant Stimuli, it was decided to break the data down into four separate sets of functions corresponding to the first, second, third, and fourth block of 100 pictures given of this group of stimuli. When this was done, the functions underwent something resembling the evolution shown in Figure 34. Initially, of course, the function would be a straight line passing through the .50 probability level for all variations of the stimulus variable. Gradually, however, it rotates with a positively accelerated characteristic. As the experiment proceeds, it continues to rotate evolving toward a negatively accelerated function. Figure 34 applies in a general way to each of the four cue variations. It is to be interpreted only in a general way since these curves are drawn merely to illustrate the effect.

In Figure 6 the results of the HS subjects are shown as they correlate with the three synthesis models. This is done for the Selected Stimuli Treatment Group based upon the criteria afforded by the Single Stimuli. It is to be observed that for the first 100 trials the Multiplicative Solution proves the best predictor. In the later blocks of trials the Additive Solution provides the higher correlations. It is also notable that the Square Root Solution begins relatively low but gains in strength until by the third block it is superior to the Multiplicative Solution. These differences are small, but it will be shown as the rest of the data are developed that these relationships are reasonably constant. These results suggest that the synthesis process varies through training. High school subjects begin by emphasizing the extreme probability values

Figure 34. Generalized probability tendency of subjects responding to the variations of a single cue. Curves are shown for each block of 100 trials. The characteristics of these functions have been magnified for purposes of illustration.



but end up after 400 trials by giving disproportionate emphasis to the central probability values. It is conceivable that if training were continued indefinitely, the Square Root Solution would prove the most precise.

Figure 7 is analogous to Figure 6 except that these results are based upon the GS results. The same effects appear to be operating except that the Square Root Solution is always inferior to the other solutions.

In Figure 8 the results for the retarded subjects are shown. Here the Multiplicative Solution remains the best predictor throughout testing. Again the differences are small, but can be shown to be consistent. Apparently the retarded subjects continue to emphasize the extreme cue values. While this procedure is mathematically logical, it produces an insensitivity to any changes in the overall probability situation.

Figures 9, 10, and 11 are of the same data shown in Figures 6, 7, and 8 except that they are assembled to compare subject sources rather than stimulus synthesis solutions. The differences are usually small and without absolute consistency. It should be remembered that these curves do not show how well the subjects performed but rather how well their performances were predicted.

Figures 12 through 17 correspond to Figures 6 through 11 except that they are based upon the Constant Stimuli rather than the Single Stimuli criteria. In general, it will be seen that the same general characteristics are present as were identified with the Single Stimuli criteria but that the correlations are lower and less discriminating between the synthesis models.

Figures 18 through 22 are analogous to Figures 12 through 17 except that they are based on the Correlated Stimulus Group. Since none of the retarded subjects were given this treatment, they are absent from these analyses.

The correlations here are somewhat larger than their counterparts on the Selected Stimuli. This is interpreted as being due to the fact that this method of association tends, in part, to relieve the subject of the necessity for synthesis. The effects noted earlier (superiority of the Multiplicative Solution displaced by the Additive Solution) are again in evidence.

Figures 23 through 27 are identical with Figures 18 through 22 except that they apply to the Random Stimulus Treatment Groups. The correlations fall between the Correlated and Selected Treatments as might be expected.

Figures 28 through 33 apply to the B+A Treatment Groups using the Single Stimulus criteria. Thus they are analogous to Figures 6 through 11. The correlations are lower, but the relationships between correlations are similar to those found for the Selected Stimuli. This seems to indicate that the method of stimulus synthesis is not affected by the number of relevant stimulus cues.

As stated earlier, these figures (6 through 33) apply only to the analyses based upon all stimulus cues (B, L, A, and H) considered simultaneously. Correlations were, however, derived for each possible combination (BLA, AH, B, etc.). These are not graphically presented since to do so would be largely repetitious to what has already been shown. They do, however, supply a means of testing the reliability of some of the effects mentioned above. This is done by comparing each correlation based upon one method of synthesis with its counterpart from another method of synthesis and tabulating the frequency that one exceeds the other. This is shown in Table 4 for the Selected Stimulus Treatment. Eleven comparisons are made from the total methods of cue combination (BL, BA, BH, LA, LH, AH, BLA, BLH, BAH, LAH, and BLAH). The numerical entries show the frequency that the correlations of the referenced analysis variable were greater than the

Table 4

Results of the Comparisons of Magnitude of Corresponding Correlations
Between the Methods of Synthesis Analysis for the Selected Stimuli

Comparisons of Methods of Analysis - 1st 100 Trials

	Add	Mult	Diff	Add	SR	Diff	Mult	SR	Diff
HS	5	6	1	8	3	5	10	1	9*
GS	3	8	5	6	5	1	11	0	11*
R	2	9	7*	5	6	1	9	2	7*

Comparisons of Methods of Analysis - 2nd 100 Trials

	Add	Mult	Diff	Add	SR	Diff	Mult	SR	Diff
HS	6	5	1	8	3	5	9	2	7*
GS	4	7	3	8	3	5	6	5	1
R	5	6	1	7	4	3	10	1	9*

Comparisons of Methods of Analysis - 3rd 100 Trials

	Add	Mult	Diff	Add	SR	Diff	Mult	SR	Diff
HS	7	4	3	8	3	5	6	5	1
GS	6	5	1	6	5	1	5	6	1
R	4	7	3	7	4	3	8	3	5

Comparisons of Methods of Analysis - 4th 100 Trials

	Add	Mult	Diff	Add	SR	Diff	Mult	SR	Diff
HS	8	3	5	8	3	5	5	6	1
GS	7	4	3	8	3	5	7	4	3
R	2	9	7*	8	3	5	9	2	7*

* $P < .06$.

corresponding correlations of the comparison variable. For individual comparisons a difference of seven or greater has less than .06 probability of occurrence $(2(\frac{1}{2})^{11} + 11(\frac{1}{2})^{11} + 55(\frac{1}{2})^{11})$. If all 36 comparisons are considered only a difference of 11 is statistically significant $(36[2(\frac{1}{2})^{11}] = .035)$.

These comparisons reinforce the conclusion that with the normal subjects at the beginning of the testing there is a slight superiority of the Multiplicative Solution over the Additive Solution and a still greater superiority of the Multiplicative Solution over the Square Root Solution. At the end of testing for these subjects, however, the Additive is superior to the Multiplicative, and the Square Root Solution has become relatively stronger. In the retarded subjects the Multiplicative Solution remains the best predictor throughout.

In Table 5 a similar comparison is made between groups. Eleven comparisons are again made from the total methods of cue combination (BL, BA, BH, LA, LH, AH, BLA, BLH, BAH, LAH, and BLAH). The numerical entries show the frequency that the correlations of the referenced group were greater than the corresponding correlations of the comparison group. For individual comparisons a difference of 7 or greater has less than .06 probability of occurrence $(2(\frac{1}{2})^{11} + 11(\frac{1}{2})^{11} + 55(\frac{1}{2})^{11})$. If all 36 comparisons are considered, only a difference of 11 is statistically significant $(36[2(\frac{1}{2})^{11}] = .035)$. The differences are generally without significance, but there is a trend which favors the normal subjects.

DISCUSSION

The significance of this study is that a difference has been uncovered in the way normal and retarded subjects integrate or synthesize the stimulus cues relating to a discrimination. The precise nature of this difference is not known.

Table 5

Results of the Comparisons of Magnitude of Corresponding Correlations
Between the Methods of Synthesis Analysis for the Selected Stimuli

Comparisons of Groups - 1st 100 Trials									
	HS	GS	Diff	HS	R	Diff	GS	R	Diff
Add	5	6	1	7*	4	3	7*	4	3
Mult	4	7*	3	5	6	1	6	5	1
SR	6	5	1	3	8*	5	4	7*	3
Comparisons of Groups - 2nd 100 Trials									
	HS	GS	Diff	HS	R	Diff	GS	R	Diff
Add	6	5	1	7*	4	3	7*	4	3
Mult	6	5	1	5	6	1	7*	4	3
SR	5	6	1	5	6	1	6	5	1
Comparisons of Groups - 3rd 100 Trials									
	HS	GS	Diff	HS	R	Diff	GS	R	Diff
Add	5	6	1	6	5	1	7*	4	3
Mult	5	6	1	5	6	1	6	5	1
SR	4	7*	3	7*	4	3	8*	3	5
Comparisons of Groups - 4th 100 Trials									
	HS	GS	Diff	HS	R	Diff	GS	R	Diff
Add	6	5	1	7*	4	3	6	5	1
Mult	6	5	1	4	7*	3	4	7*	3
SR	6	5	1	9*	2	7*	8*	3	5

* $P < .06$.

The Additive, Multiplicative, and Square Root Formulas only represent tentative and preliminary solutions to the problem. They are, as it were, guide posts to indicate the direction toward greater precision. It would be of much less significance had one of these formulas simply turned out to be the best predictor for all groups and all stages.

In common with the retarded subjects, the normals begin a type of synthesis that is best represented by the Multiplicative Solution. Mathematically this is the most logical approach to the problem. Unlike the retarded, however, the normals seem to drift away from it to a type of synthesis that is best described by the Additive Solution. The Square Root Solution also becomes relatively more precise in the case of the Normal Groups. The reader is cautioned not to infer that overall performance degraded through time, it is only that the method of stimulus synthesis became less precise based upon mathematical criteria. This undoubtedly affects overall performance, but since other factors are involved (especially single cue discrimination) total performance improves. It is interesting to speculate on these results. It is difficult to understand why normal subjects should turn to a less efficient synthesis process while the retarded persevere in the superior procedure. Then again, the human organism may ultimately be more rational than we imagine. It is true that for a problem whose parameters are pre-established and unalterable, the Multiplicative Solution produces the most consistent reinforcement. Computers programmed to accept problems of the kind described here are, of course, much more efficient than men. The human, however, has been around a long time. It may be that his particular process is, in the long run, more efficient since it is more sensitive to changes in the environment. Man is an excellent general computer due to his flexibility and adaptability. By continuously

sampling from low probability hypotheses he retains a degree of versatility and sensitivity that the more precise computers do not enjoy.

As it relates to precision, there are obvious defects in the present study. Although the continuity-noncontinuity question has been hotly debated there seems to be no question that in human learning, at least, the subject selects those stimuli and those hypotheses to which he responds. One has only to examine an individual record to see that in the early stages of testing the subjects alternate between response possibilities. This being the case, it follows that only those hypotheses which have been formulated at the moment of response are affected by the consequences of that response.

In the present analysis no effort was made to distinguish between the many different stimulus selection solutions as they might be used by the individual subjects. This has only been done on a group basis. Thus, one subject might be responding to some combination of B and L while a second subject is responding to a combination of A and H. Their results are, however, combined which undoubtedly affects the sensitivity of the analysis. This procedure is, however, justified for these preliminary studies. As the general process becomes more evident, methods of analysis will be developed to account for the variance due to the individual differences.

STUDY NO. 2

STIMULUS SYNTHESIS FOLLOWING A QUANTITATIVE RESPONSE AND KNOWLEDGE OF RESULTS

In Study No. 1, the data were taken in a form that could be applied toward the analysis of various hypotheses or probability models. In this sense it was inductive in its approach and was intended only to serve as a starting point for the project. The present experiment is more accurately described as deductive in nature because it evaluates three different hypotheses, and the utility of the data is limited to these evaluations.

Despite this limitation, there are advantages to the present study. This arises primarily from the fact that while Study No. 1 was based upon a binary choice (X or Y), the present study allowed for a quantitative or scalar response.

These differences, notwithstanding, the present study relates to the first in that the hypotheses tested are similar to those evaluated in the earlier experiment. Since the procedures and methods of analysis differ, the results of the second experiment provide a method of validating the conclusions of the earlier work. This follows from the consideration that the results from any given study may not be of general application, but rather merely reflect the experimental method and the analysis used. Proceeding from this, if one can alter the method and analysis and still reach similar conclusions, then there is some evidence that the models are of general rather than specific application.

In common with the previous study, the purpose of this experiment was to compare various hypotheses concerning the method by which subjects integrate

stimulus cues. This was accomplished by comparing three different treatments. The treatments differed only in terms of the type of knowledge of results given after each response.

In a gross sense knowledge of results reinforces or extinguishes a habit. More precisely, however, it operates in a manner analogous to a servo system by correcting too great for too small a response. It thus operates as that link in a closed loop circuit which feeds back information to the subject concerning the appropriateness of his response. Let it be supposed that the human method of integrating diverse stimulus cues follows a systematic and generally unalterable procedure of weighting and combining the stimulus elements. It then follows that the feedback system which most nearly corresponds to the perceptual process will produce the most rapid learning and the most stable results. If the feedback process is altogether different from the natural perceptual process, continuous instability will result, since one will never be in agreement with the other. If the two processes are identical, however, the magnitude of the response and return signal will correspond perfectly.

METHOD

Subjects

One hundred fifty-two subjects were obtained from the public schools, 76 from a high school (HS), and 76 from a grade school (GS). The HS students ranged in age from 15 to 18 and the GS students from 9 to 11. Within each age level, subjects were randomly assigned to one of three treatment groups. A total of 152 subjects completed the task. Before the data analysis, 32 subjects were discarded

either because they failed to respond to each trial or because they delayed their answers until after knowledge of results (KR) had been given. Subjects were paid for their participation, the amount depending on performance.

Stimuli

The stimuli used were the 400 photographic transparencies of the Selected Stimulus Group described earlier. An example of these is seen in Figure 1. Four cues were available, each having twenty variations. They were the position of the arms (Cue A), position of the legs (Cue L), position of the head (Cue H), and the length of a bar above the girl's head (Cue B). Each cue variation appeared twenty times in selected combinations with the other cues.

It was established arbitrarily that as the girl's head turned left, as her arms came up, as her legs came together, and as the bar lengthened, the composite picture increased in value. For example, as seen in Figure 3, the length of the bar in the shortest variation was given a cue value of one. In variation twenty, the longest length, it was given a value of twenty. The same relationship holds for the arms, legs, and head. There was a linear correspondence between variations of the cues and cue values.

Procedure

The slides were randomized with respect to order of presentation and shown by means of a 35 mm projector and a 3' x 3' screen. The subjects, in groups of ten or less, were seated in clear view of the screen with pens and data sheets. The procedure involved the presentation of a slide, requesting subjects to evaluate it with a score from 0 to 100, and, after all had responded, giving KR

concerning the value of the slide as determined from one of the three hypotheses. Thus the subjects, beginning with no attitude about the value of the cues, could gradually bring their evaluations into correspondence with the KR given. Since the subjects were tested in small groups and closely monitored, it was not difficult to insure that answers were recorded prior to delivery of KR. In those cases where it appeared an effort was being made by some subjects to circumvent this regulation, a stratagem was employed to check the data. This consisted of having subjects pass pens to predesignated neighbors immediately after responding. In the circular route taken by the pens, it was arranged so that the ink color would alternate--red, blue, green, red, blue, green, etc. Later, by inspection of the data sheets, it was possible to insure that answers were recorded prior to the KR given.

The HS and GS subjects were each divided into three treatment groups. Otherwise, all subjects were tested alike. Those treatment groups differed in the kind of KR given at the termination of each trial. The first treatment can be designated as the Additive Hypothesis Treatment. The second and third are the Square and Log Hypotheses, respectively. The KR given the Additive Treatment Groups was determined by the following formula:

$$V_A = (B-10 + L-10 + A-10 + H-10 + 36) 1.39.$$

The letters B, L, A, and H indicate the assigned values shown in Figure 3. V_A represents the additive value of a slide. The remaining manipulations were performed to produce a scale ranging from 0 to 100. That is, the adding of 36 brought all numbers to a positive value, while 1.39 was the value needed to produce the 0 to 100 scale.

For the second treatment condition, KR was based upon squaring the values of the cues. The formula used is shown below:

$$V_S = (|B-10|^2 + |L-10|^2 + |A-10|^2 + |H-10|^2 + 324) .154.$$

This also has a scale ranging from 0 to 100. This formulation tends to give proper emphasis to the extreme values at the expense of those values more centrally located. This function is similar, but not so extreme, as the Multiplicative Hypothesis tested earlier.

The final treatment condition was obtained from the following formula:

$$V_L = (\log B-10 + \log L-10 + \log A-10 + \log H-10 + 4) 12.5.$$

This equation produces a function which tends to emphasize central as opposed to the extreme values. This equation is similar, but again not so extreme, as the Square Root Formula examined in the preceding experiment.

The difference between these functions can be appreciated from inspection of the curves shown in Figure 35. These were produced on the basis of a purely theoretical association of stimulus variations. As such, they do not represent any actual stimuli used, but only show the scalar trend which develops when the individual stimulus values are concomitantly increased. It can be seen that the rate of change for the linear function is constant. The square function shows its greatest rate of change at the extremities, while the log function shows the greatest change near the middle values. These relationships are reflected in Figure 36 which shows the plot obtained from a random sample of the stimuli used. While the points illustrated in the figure are random variations, the basic S shape is still to be discerned in the log function and the inverted S for the square function.

Figure 35. The theoretical distribution of the linear, log, and square slide values compared with the linear slide values. The curves were obtained by using certain systematically varied cue values which were not found in the stimuli actually used.

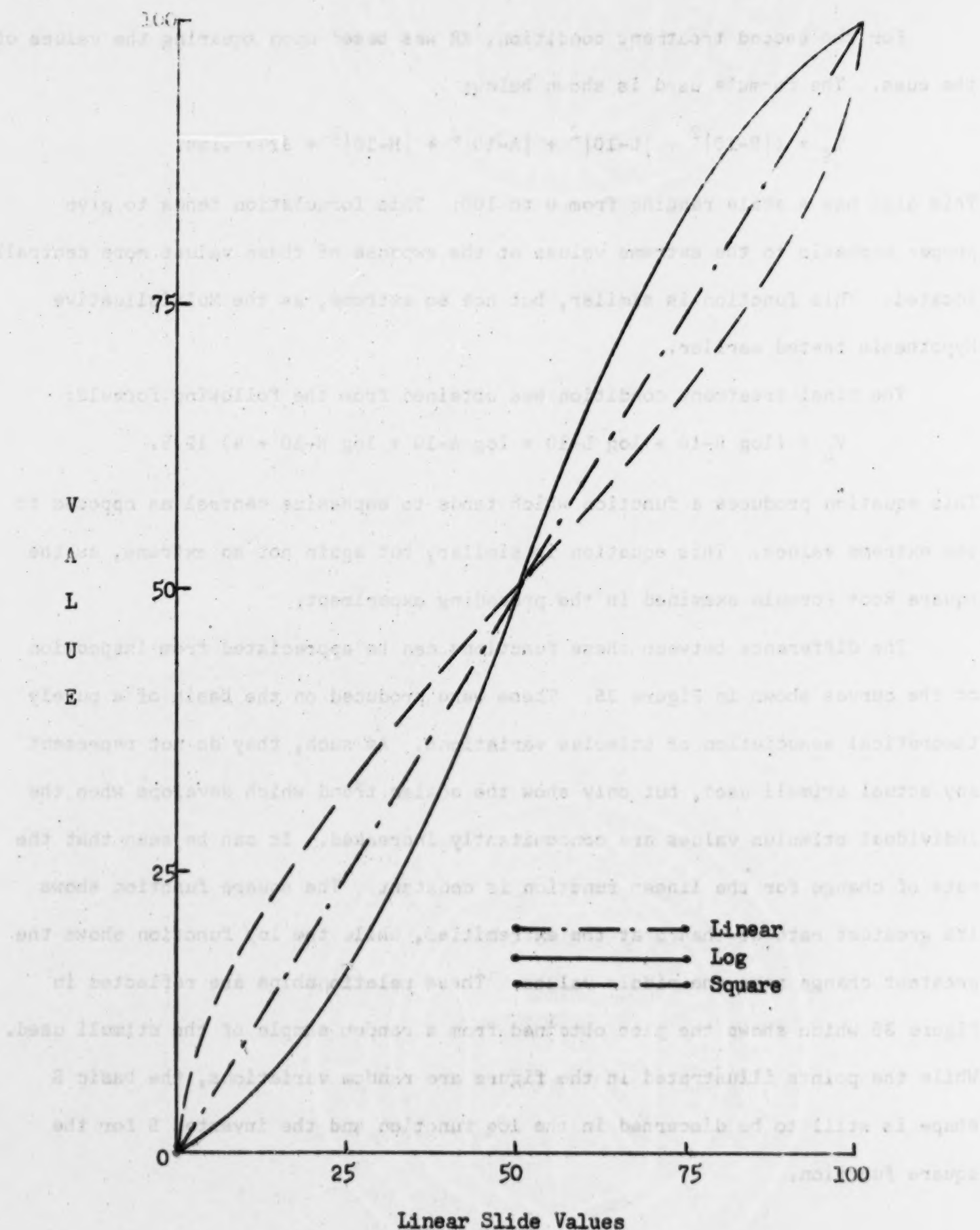
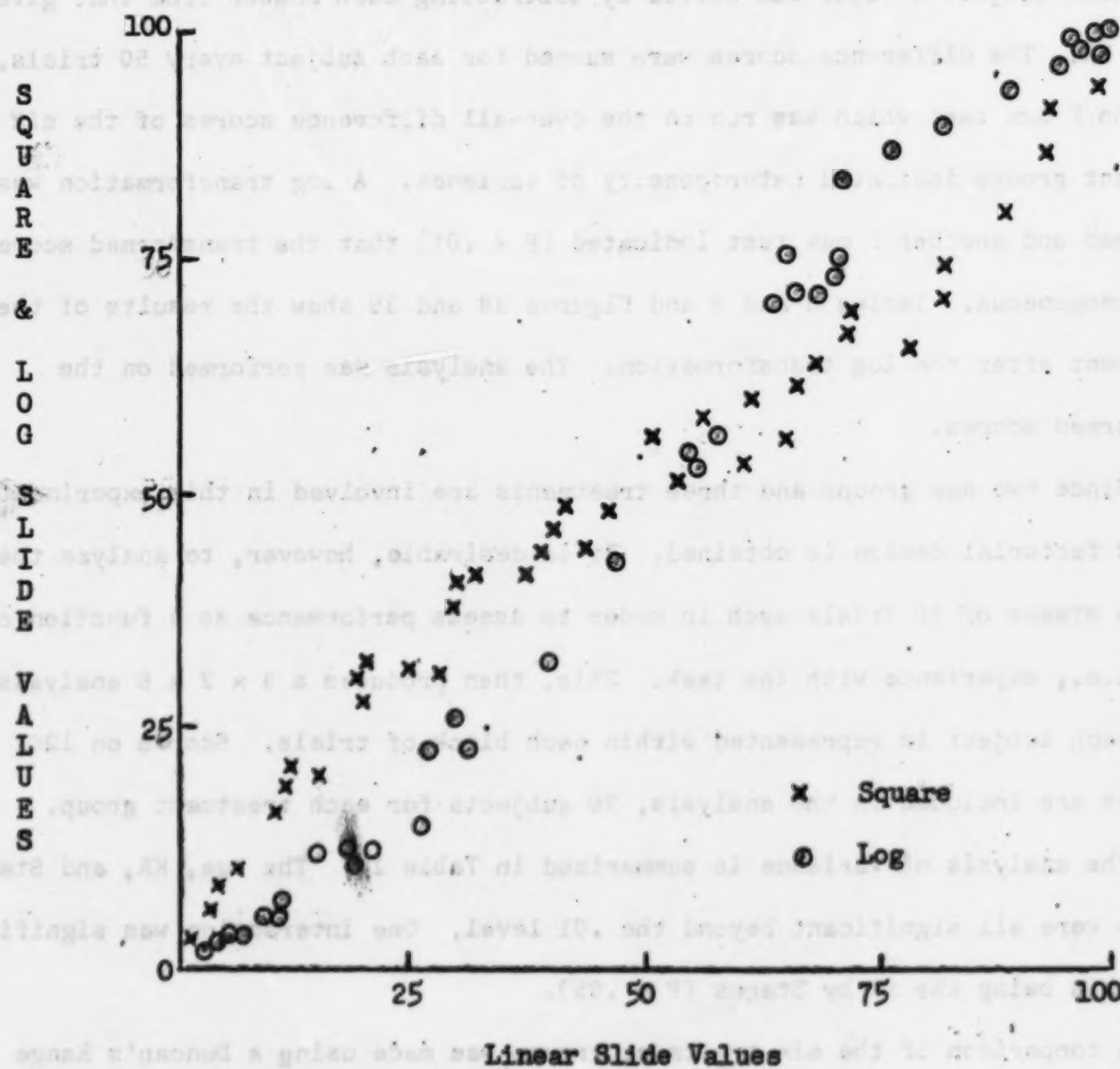


Figure 36. A sample of the square and log value of slides actually obtained compared with the linear values of the same slides.



RESULTS

The results of the experiment are shown in Tables 6 and 7 and Figure 37. From these tables and figures it can be seen that the analysis of the task was broken down into eight blocks (stages) of 50 trials each.

Each subject's paper was scored by subtracting each answer from that given to him as KR. The difference scores were summed for each subject every 50 trials.

An F max test which was run on the over-all difference scores of the six treatment groups indicated heterogeneity of variance. A log transformation was performed and another F max test indicated ($P < .01$) that the transformed scores were homogeneous. Tables 8 and 9 and Figures 38 and 39 show the results of the experiment after the log transformation. The analysis was performed on the transformed scores.

Since two age groups and three treatments are involved in this experiment, a 3×2 factorial design is obtained. It is desirable, however, to analyze the data in stages of 50 trials each in order to assess performance as a function of time, i.e., experience with the task. This, then produces a $3 \times 2 \times 8$ analysis in which each subject is represented within each block of trials. Scores on 120 subjects are included in the analysis, 20 subjects for each treatment group.

The analysis of variance is summarized in Table 10. The Age, KR, and Stage effects were all significant beyond the .01 level. One interaction was significant, this being the KR by Stages ($P < .05$).

A comparison of the six treatment groups was made using a Duncan's Range Test (Table 11). Each treatment group was significantly different from all others at the .001 level.

Table 6
Mean Error of Treatment Groups (N = 20) by Stages

		Stages							
		1	2	3	4	5	6	7	8
GS	Log	1237	870	859	777	882	779	704	593
	Square	1042	812	713	693	694	653	595	565
	Add	1109	855	701	596	659	589	562	441
HS	Log	989	631	633	556	612	560	549	413
	Square	848	614	558	493	512	480	413	406
	Add	789	496	471	448	494	467	433	373

Table 7
Means and Standard Deviations of Treatment Groups (N = 20)
on Over-all Error

	HS		GS	
	$\bar{X}$	SD	$\bar{X}$	SD
Log	4942.700	1913.181	6700.900	2617.539
Square	4323.500	1665.938	5767.050	2225.559
Additive	3970.150	1517.798	5513.150	2217.467

Figure 37. Mean error of the six treatment groups (N = 20) as a function of stages (blocks of 50 trials).

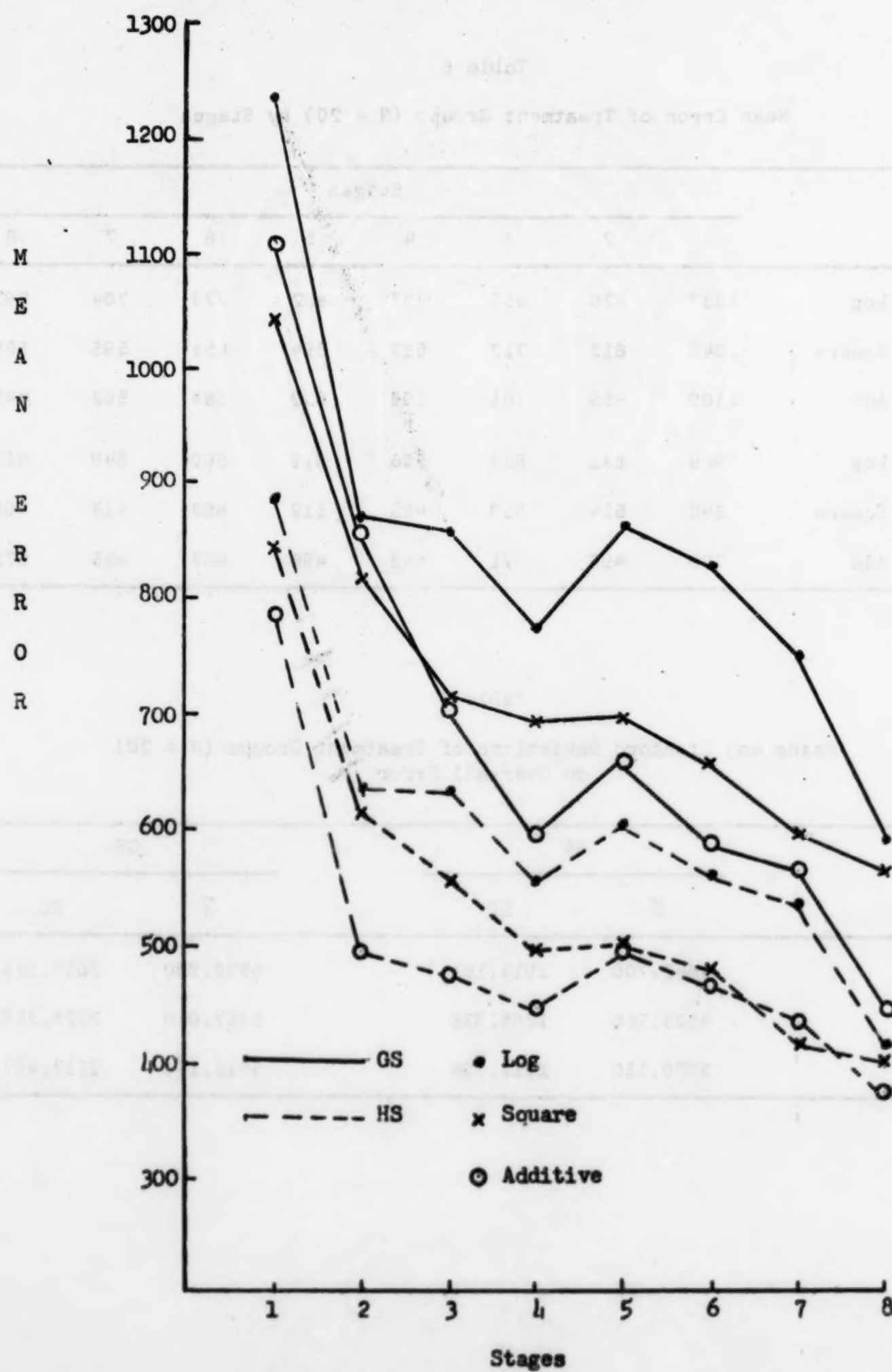


Table 8
Means and Standard Deviations of Treatment Groups (N = 20)
on Over-all Error of Transformed Data

	HS		GS	
	X	SD	X	SD
Log	6.115	.212	7.139	.284
Square	5.662	.219	6.605	.263
Additive	5.388	.216	6.364	1.072

Table 9
Mean Error by Stages of Treatment Groups (N = 20)
on Transformed Data

	Stages							
	1	2	3	4	5	6	7	8
Grade School								
Log	1.067	.909	.914	.869	.932	.864	.839	.746
Square	1.007	.887	.829	.817	.824	.799	.751	.740
Add	1.009	.879	.814	.761	.803	.746	.730	.662
High School								
Log	.976	.777	.790	.735	.771	.735	.727	.604
Square	.905	.776	.732	.680	.696	.665	.607	.600
Add	.875	.686	.662	.644	.681	.650	.625	.557

Figure 38. The mean log error of the six treatment groups (N = 20) as a function of stages (blocks of 50 trials),

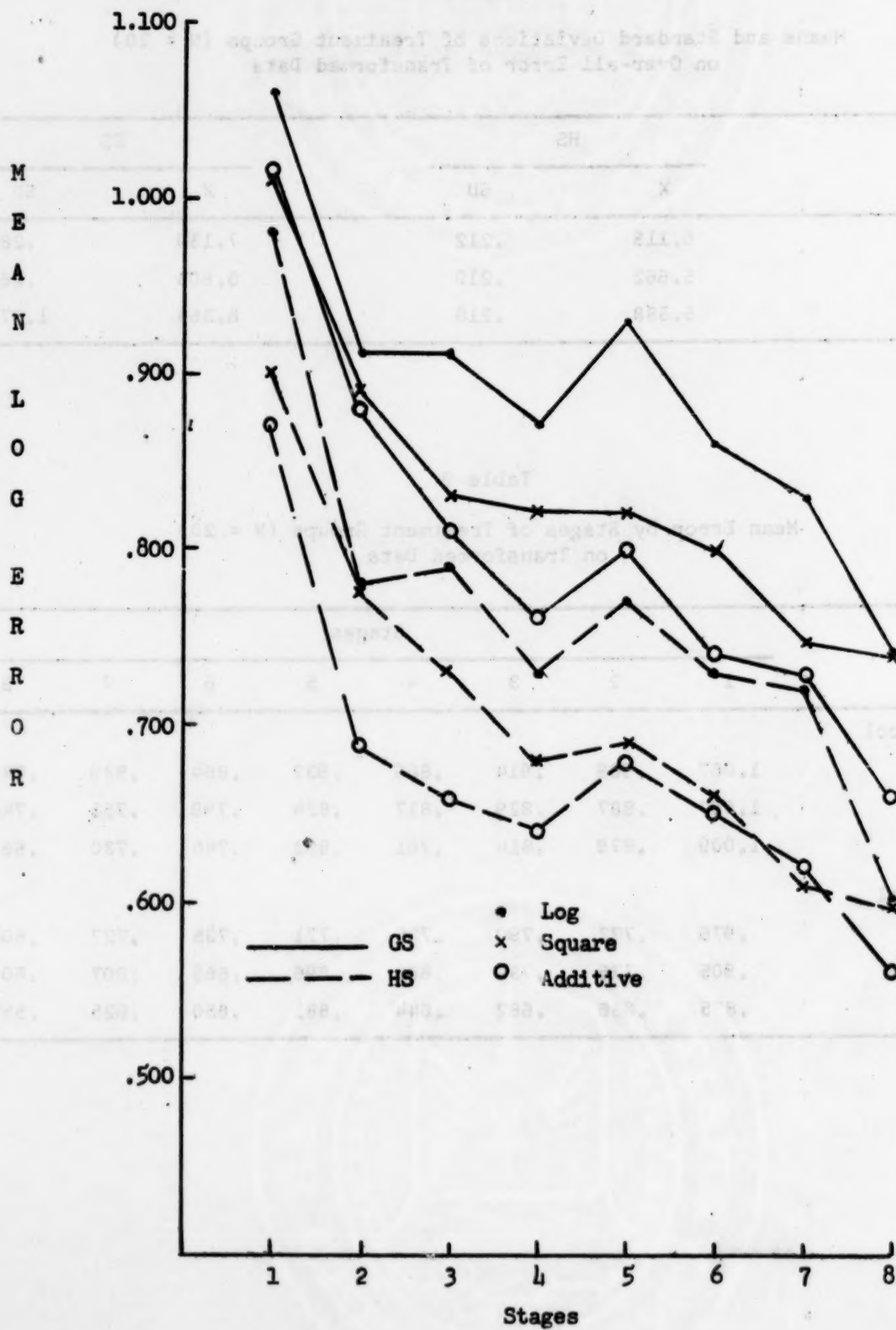


Figure 39. The mean log error of the KR groups (N=40) summed over age by stages (blocks of 50 trials).

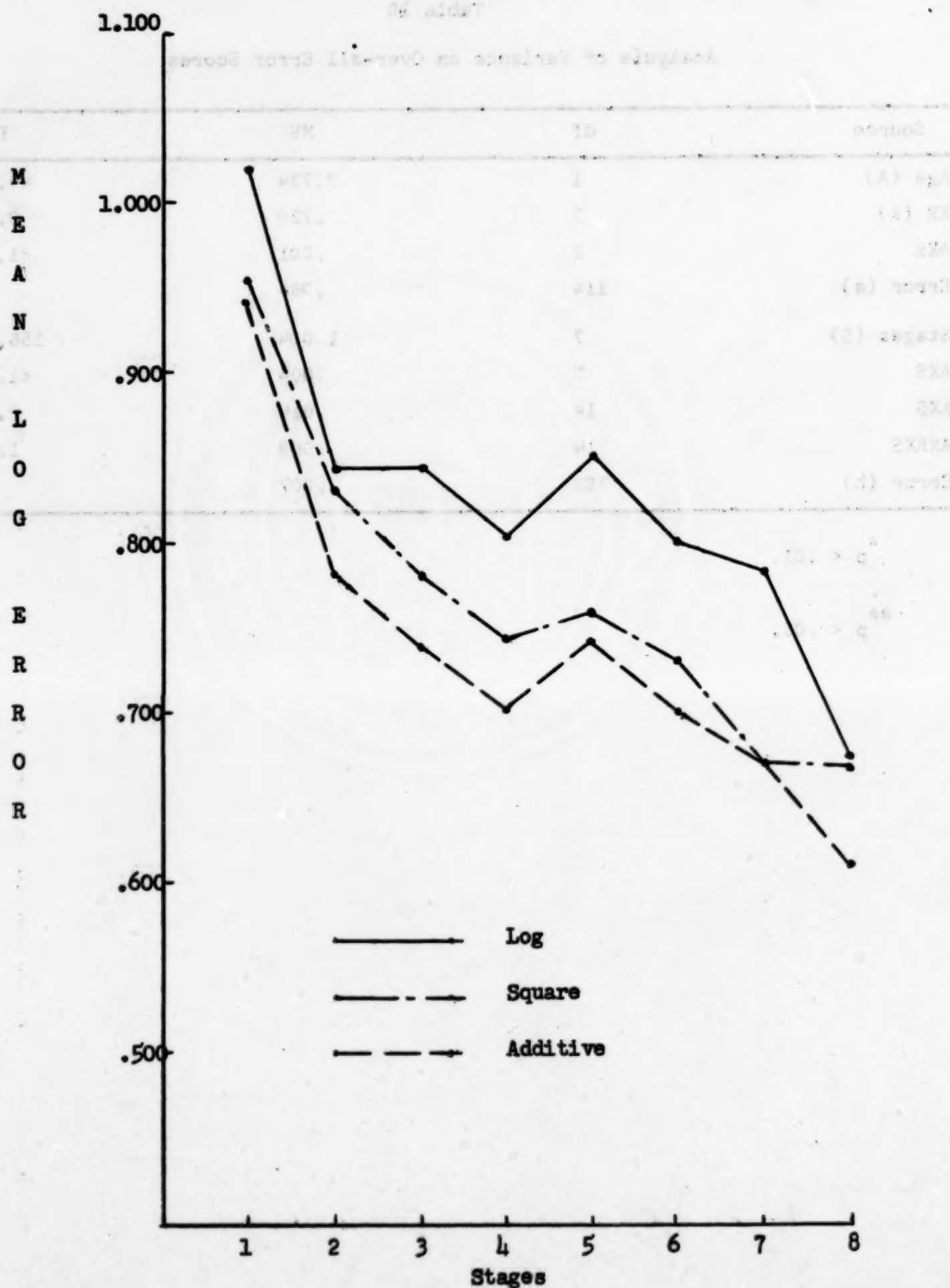


Table 10

Analysis of Variance on Over-all Error Scores

Source	df	MS	F
Age (A)	1	3.734	44.45*
KR (K)	2	.720	8.57*
AXK	2	.001	<1.00
Error (a)	114	.084	
Stages (S)	7	1.094	156.29*
AXS	7	.004	<1.00
KXS	14	.014	2.00**
AXKXS	14	.009	1.29
Error (b)	799	.007	

* $p < .01$.** $p < .05$.

Table 11

Duncan's Range Test on Over-all Performance

		GS-Log	GS-SQ	GS-Add	HS-Log	HS-SQ	HS-Add	Shortest Significant Ranges
	Mean	7.139	6.605	6.364	6.115	5.662	5.388	
GS-Log	7.139		.534	.775	1.024	1.477	1.751	$R_2 = .206$
GS-SQ	6.605			.241	.490	.943	1.217	$R_3 = .212$
GS-Add	6.364				.249	.702	.976	$R_4 = .217$
HS-Log	6.115					.453	.727	$R_5 = .220$
HS-SQ	5.662						.274	$R_6 = .223$
HS-Add	5.388							

All means significantly different at $P < .001$.

GS = Grade School; HS = High School; Log = Log KR; SQ = Square KR; Add = Additive KR.

Another Duncan's Range Test was run on the means of the KR Groups (Table 12). The KR Groups all differed beyond the .001 level.

Table 12

Duncan's Range Test on Comparison of KR Means

		Log	Square	Add	Shortest Significant Ranges
	Mean	6.627	6.131	5.876	
Log	6.627		.496	.751	$R_2 = .125$
SQ	6.131			.255	$R_3 = .126$
Add	5.876				

All means significantly different at $P < .001$.

The results show that the HS group performed with less error than the GS group. The Linear KR Groups performed with less error than the Square Groups which in turn performed better than the Log Groups. This latter difference was also noted with the KR Groups summed over age.

DISCUSSION

It was assumed in this experiment that subjects receiving the kind of KR which was most compatible with the kind of stimulus synthesis process human organisms employ would perform with less error. That is, it was expected that by giving different kinds of KR to different treatment groups, the group or groups receiving KR most congruent with the manner in which humans evaluate and combine cues from their environment would be superior in performance.

In general the results of the present study tend to support the previous experiment. The analysis of the performance scores of the Additive, Square, and Log Treatment Groups shows that the Additive Group was superior in performance to the Square and Log Groups and that the Square Group performed with less error than the Log Group.

The significant age effect can be attributed in part to superiority on the part of HS subjects in the ability to synthesize. In addition, HS students have a much better grasp of number concepts. All the HS groups performed with less error than any of the GS groups (Table 11). Within each age group, the KR treatments are consistent, i.e., for both age groups, the Additive Groups had the least error followed by the Square and Log Groups, respectively. The similarity of performance is borne out by inspection of Figure 38.

From inspection of the means (Table 9 and Figure 38) it is clear that the amount of error decreased as experience with the task increased; that is, learning occurred. This accounts for the significant stage effect.

The one significant interaction obtained, KR by Stages, appears to be attributable to the early flattening of the log curve (Figure 39). The Log KR Groups showed little decrement from stages one to seven, while the Square and Additive KR Groups continued to improve their performance.

The proposed statistical treatment originally included a trend analysis similar to the analysis described by Grant (1956). The analysis was conducted, but as might have been expected from inspection of the curves (Figure 38), it was essentially inconclusive. This figure shows that the slopes of the curves for the treatment groups were very similar.

While the results strongly indicate that a Linear model comes closest among the hypotheses tested to describing the stimulus synthesis process, this result should not be interpreted to mean that the Linear model described it precisely. In fact, it is best to conclude that a model somewhat between the Additive and Square functions better describes the synthesis process. This conclusion is based on the fact that except at the extremes and in the middle where the functions meet, the Linear model tends to lie half way between the Log and Square models. The Square Hypothesis, however, is a better "fit" to the synthesis process than the Log Hypothesis. It must be concluded, then, that the actual function falls somewhere between the Square and Additive models because if this were not so, there would not be a significant difference between the Square and Log Groups.

An even closer description of the synthesis process would have to take into account a great many factors. For instance, one of the factors very likely

operating in this task is stimulus generalization. If subjects receive positive reinforcement for a cue value, the effect of the reinforcement should generalize to the adjacent cues. These cue values would acquire some reinforcement, then, even though they were not directly reinforced on that trial.

No doubt other factors influence performance on this task. The problems of identifying them and finding appropriate methods of analysis will be the subject of continued research.

The negative acceleration of the error curves (Figure 38) did not reach an asymptote. Learning was still occurring at the conclusion of the experiment. From this observation and because all the KR Groups ended very closely together, it can be assumed that certain changes in the synthesis process occurred as a function of experience with the task. The Log and Square Groups apparently adopted means of solving the problems more effectively as time progressed.

The experiment bears out the implications of previous research. The Linear and Square Hypotheses of stimulus synthesis appear to be the best description with the Linear being somewhat closer than the Square. Further research is dictated to determine some of the other factors entering into the problem.

STUDY NO. 3

THE EFFECT OF STIMULUS GENERALIZATION ON CUE INTEGRATION

The preceding studies served the purpose of indicating the general trend in stimulus synthesis. While not perfunctory in their analyses, they nevertheless left many avenues unexplored. This is justified in view of the complexity of the processes and our ignorance concerning them. As more is learned, however, greater refinement must be applied to procedures and analyses.

The perception of the subject is dependent upon a number of factors. Chief among these, as far as this project is concerned, are the method of stimulus synthesis and the intelligence of the subject. We have seen that these interact with one another and that both interact with amount of training.

A subject's performance, relative to some criteria or theory, can be thought of as an aggregate of variances contributed by the imprecision of the theory and by the other uncontrolled or unaccounted for effects.

In attempting to cast a model in description of a given effect additional refinement very quickly runs into the law of diminishing returns. This is particularly true when the variance which remained as a consequence of any lack of precision in the theoretical model is small relative to the variance contributed by the uncontrolled and extraneous variables.

It is evident, for example, that in the discriminative paradigm employed in these studies more is involved than the level of intelligence and synthesis. Some of these may be of small importance and can be ignored with impunity. Others,

acting in concert, may produce a great deal more variance than the effects that are the principle subject of investigation. This being the case, it behooves the student to account for as much of this extraneous variation as possible in order to systematize his calculations.

This procedure, of course, imposes difficulties. First, it is difficult merely to list all of the relevant effects. Second, many of these effects are no better understood than the effects that are of primary concern, and it is impractical to embark upon a program designed to obtain a precise description of each effect as it interacts with the other factors.

Happily, for this stage of the program's development, it is neither necessary to account for all of the variables nor to describe them with absolute precision. If we are interested in effect A, which is obscured by effects B, C, and D, we can greatly improve our estimate of A by accounting for only a portion of the variance contributed by B, C, and D.

In the present study an attempt is made to account for two of these "extraneous" variables. These are rate of learning and stimulus generalization. In one sense, cognizance was taken of these factors in the first study by the use of special groups of subjects (orders B, L, A, and H) whose results were used as criteria for the experimental groups (Selected, Correlated, Random, and B+A). This procedure was indirect, however. Moreover, it was based upon the assumption that the irrelevant cues were without effect. It is not likely that this is the case, although it was convenient at the time to assume so.

In the present experiment no account was taken of the individual subject's selection of cues or of his guessing habits, although these considerations are certainly germane to a complete analysis. It is, moreover, assumed that knowledge

of results, whether confirmatory (reinforcing) or nonconfirmatory to a response, is of equivalent effect. This is almost certainly not the case with the retarded, and it is of doubtful application in normals. To take a simple example, suppose a subject is operating on the hypothesis that when the arms are in position P the appropriate response is X. If this response is confirmed there undoubtedly is a growth in the tendency of the subject to respond with an X on a subsequent appearance of the stimulus variation. This is due to the fact that the hypothesis stated above has been strengthened. Suppose on the other hand the response has not been confirmed. It does not necessarily follow that the opposite hypothesis (when the arms are in position P the appropriate response is Y) is thereby confirmed or even affected since it must compete with other possible hypotheses such as (1) response X is usually reinforced in the presence of position P or (2) the position of the arms is irrelevant to the X and Y response categories, etc.

These are variables of which some account must be taken before it will be possible to describe stimulus synthesis with the necessary degree of precision. The approach taken here is a matter of successive approximation by clearing away as much of the superfluous sources of variance as is practical before proceeding to the next level of description.

METHOD

Subjects

These consisted of 28 High School students (HS), 20 Grade School students (GS), and 61 Retarded subjects (R). The HS group was obtained from two public schools. They were sophomores and juniors with a mean age of 16.5. The GS group

was also obtained from two public schools and had a mean age of 10.5 being from the fifth and sixth grade levels. The R subjects were obtained from a public institution for the mentally retarded. Their mean IQ was approximately 55 with a range of perhaps ± 15 .

Stimuli

These consisted of the 400 pictures of the Selected Stimulus Group described in Study No. 1. The parameters are shown in Figures 3 and 5.

Procedure

This was identical with Study No. 1 except that no use was made of the Constant Stimuli. Each picture was shown in a pre-established order, and the subjects responded as an X or a Y (the normals wrote their answer on data sheets while the retarded pushed either the X or the Y switch). After the response was recorded, knowledge of results was given.

RESULTS

The analytical procedure was basically one of constructing probability models from our own observations and from the psychological literature. These were applied against the results. Reduced to a brief description, there were three generalization hypotheses (G_L , G_S , and G_H) and five synthesis hypotheses (I_S , I_G , I_L , I_M , and I_{Log}) produced in combination with one another. These were applied against the three groups of subjects (HS, GS, and R) for each of the four blocks of 100 trials. This produced 180 correlations which were used as the principal data of this study.

These were obtained in three stages of analyses. In the first stage it was necessary to determine the strength of each variation (20), of the cues (4), for the generalization hypotheses (3), for the number of trials (400), for both X and Y (2). This produced a requirement for 192,000 solutions.

The synthesis hypotheses (5) were applied in the second stage of analysis against each generalization hypothesis (3) for each trial (400). This procedure generated 6,000 solutions.

In the third phase these solutions were divided into blocks of 100 trials producing 60 sets of predictions. Correlating each of these with the empirical data of the three groups of subjects obtained the 180 correlations.

A more detailed account follows.

It was assumed that each stimulus variation of each cue had a certain potential to elicit an X response and that this potential could be represented numerically. It was also assumed that at the beginning of testing these values were small and equal to one another.

An identical set of assumptions were made regarding the elicitation of the Y responses.

It was further assumed that as a consequence of knowledge of results following a response the potential of all variations to elicit the confirmed response (X or Y) was increased as a function of their similarity to the stimulus variation so confirmed. Quite simply, if the arms in position 18 were confirmed to be a Y then position 18 increased its potential to elicit Y by a given amount. At the same time positions 17 and 19 also increased their potential to elicit Y but to a somewhat lesser degree, etc. Thus some kind of a stimulus generalization function was created. Finally, it was also assumed that there was no

diminishment in the absolute potential of these variations to elicit the X response although they did, of course, suffer diminishment relative to the Y response.

Continuing with the above example, the increase in the potential of position 18 to elicit Y as compared to the increase in positions 17, 16, etc., is made subject to differential analysis. For this purpose three generalization functions were evaluated. The first was linear and had the appearance shown in Figure 40. It was written as

$$G_L = R - (S_R - S_G).$$

Where R is the stimulus range as measured by the number of subjectively equal stimulus variations (in the present instance, 19), S_R is the number assigned to the confirmed stimulus variation and S_G the number assigned to the stimulus variation being affected. Thus if the arms were shown in variation 16 and this picture was subsequently identified as an X then this variation grew by 19 points. Variations 15 and 17 grew by 18 points, etc. This is shown in Table 13.

The second formula is similar to a function proposed by Spence (1937), to account for transposition and has the form

$$G_S = \sqrt{R^2 - (S_R - S_G)^2}.$$

The terms are as defined above. This produces the positively accelerated decay function shown in Figure 40. The relative growth of the stimulus variations using this formula is also shown in Table 13 for a confirmation of variation 16 to be an X.

The third generalization hypothesis is an approximation obtained by Hoveland (1937). It is shown as the negatively accelerated curve in Figure 40 and has the formula

Figure 40. Generalization functions G_L , G_S , and G_H used in the theoretical models.

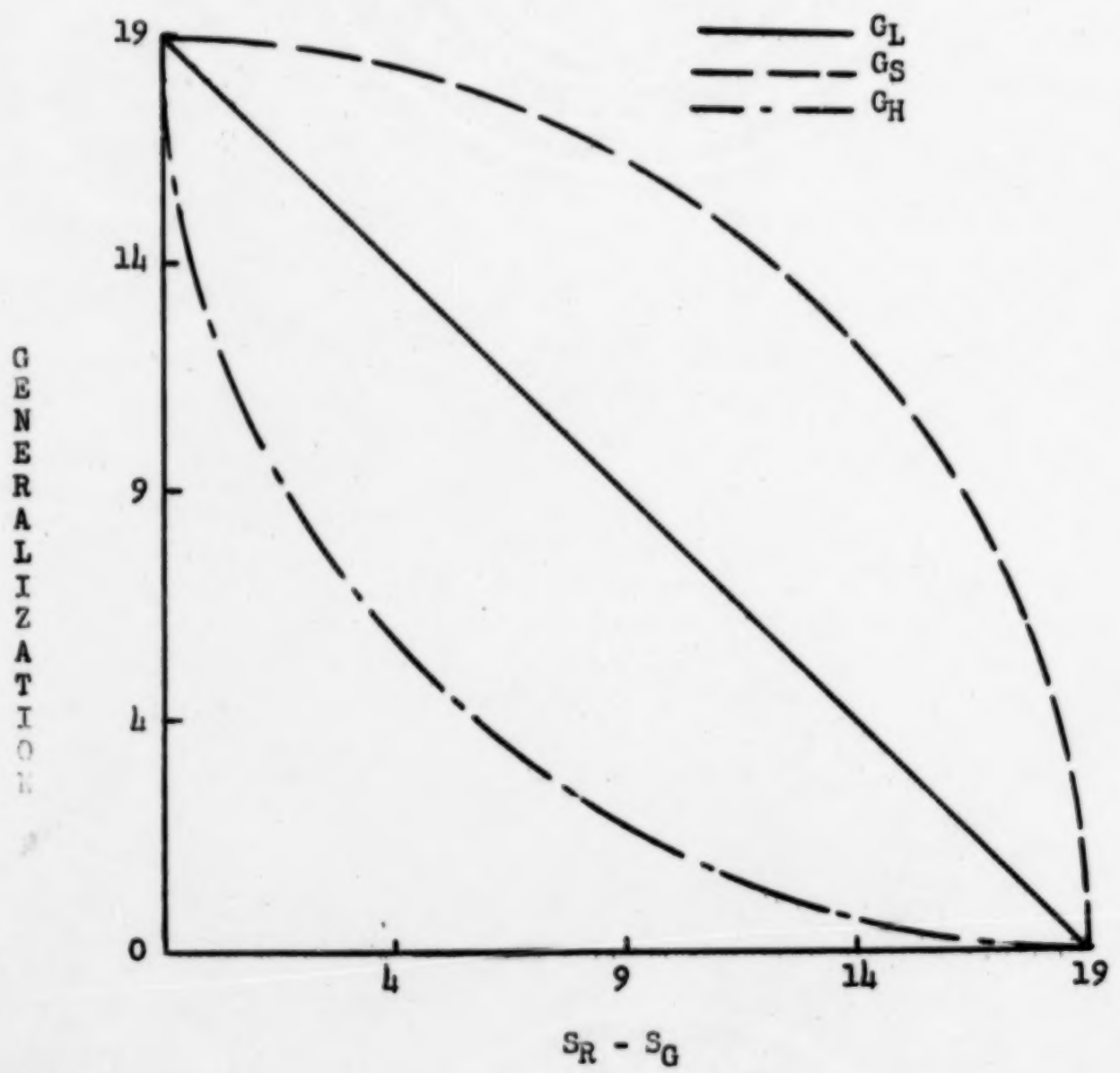


Table 13

The Relative Growth for a Given Trial of Each Variation of the Arms
with Respect to Their Potential to Elicit an X Response
Assuming that Variation 16 is Confirmed to Be an X

Variation	G _L	G _S	G _A
1	4	11.66	.43
2	5	12.84	.67
3	6	13.85	.98
4	7	14.73	1.34
5	8	15.49	1.77
6	9	16.15	2.27
7	10	16.73	2.85
8	11	17.23	3.51
9	12	17.66	4.27
10	13	18.02	5.15
11	14	18.33	6.16
12	15	18.57	7.34
13	16	18.76	8.76
14	17	18.89	10.52
15	18	18.97	12.92
16	19	19.00	19.00
17	18	18.97	12.92
18	17	18.89	10.52
19	16	18.76	8.76
20	15	18.57	7.34

$$G_H = R - \sqrt{R^2 - [R - (S_R - S_G)]^2}$$

Again the terms are as defined above and the relative growth of the variations for a single confirmation of variation 16 as an X is shown in Table 13.

A factor other than stimulus generalization in the growth of the potential of a stimulus to elicit a response is the ratio of its magnitude to its theoretical limit. The evidence afforded by most learning curves (cf. Estes, 1953) suggests that the growth of a habit is inversely related to the momentary

probability of occurrence. Let it be supposed that at a given moment in the history of an organism the probability of Stimulus S eliciting response R_1 is P_1 and that its probability of evoking response R_2 is P_2 and that P_1 is greater than P_2 . If S is presented and R_1 is confirmed as appropriate (reinforced) then P_1 is increased by Δ_1 . If, on the other hand, R_2 is reinforced (or otherwise confirmed) P_2 is increased by Δ_2 . It follows from the earlier statement that $\Delta_2 > \Delta_1$.

The change in the probability of the occurrence of a response is not a single inverse function of its momentary probability, however. The rate of change attenuates as training progresses. Quite aside from any consideration of momentary probability, reinforcing events become less and less effective in modifying behavior as training proceeds.

Habit operates as though it had weight or mass. In short, it exhibits inertia and becomes more and more difficult to displace as experience accumulates. This potential which arises out of the accumulated experience is symbolized as D.

D_X is an index of the potential of the stimulus to elicit the response X while D_Y denotes its potential to elicit response Y. At the beginning of testing D_X and D_Y are equal and of low value.

The momentary probability of the subject responding with an X on the nth trial is

$$P_{X_n} = \frac{D_{X_n}}{D_{X_n} + D_{Y_n}}$$

At the beginning of training this will be very near .50 as will P_Y since $P_Y = 1.00 - P_X$. If on the nth trial the stimulus is confirmed to be an X, the probability of all stimuli to evoke the X response is raised in the following manner

$$\Delta_{X_n} = G_{X_n} P_{Y_n}$$

Where delta is the growth, G is the generalization factor (which differs depending upon which of the three generalization formulas is used) and P_Y is the probability of the Y response (the inverse function of P_X). Thus D_X changes because

$$D_{X_{n+1}} = D_{X_n} + \Delta_{X_n}$$

The new probabilities can then be recomputed.

In this manner P_X and P_Y were obtained for each of the 20 stimulus variations of the four cues for each trial of the experiment. These were used in the synthesis formulas.

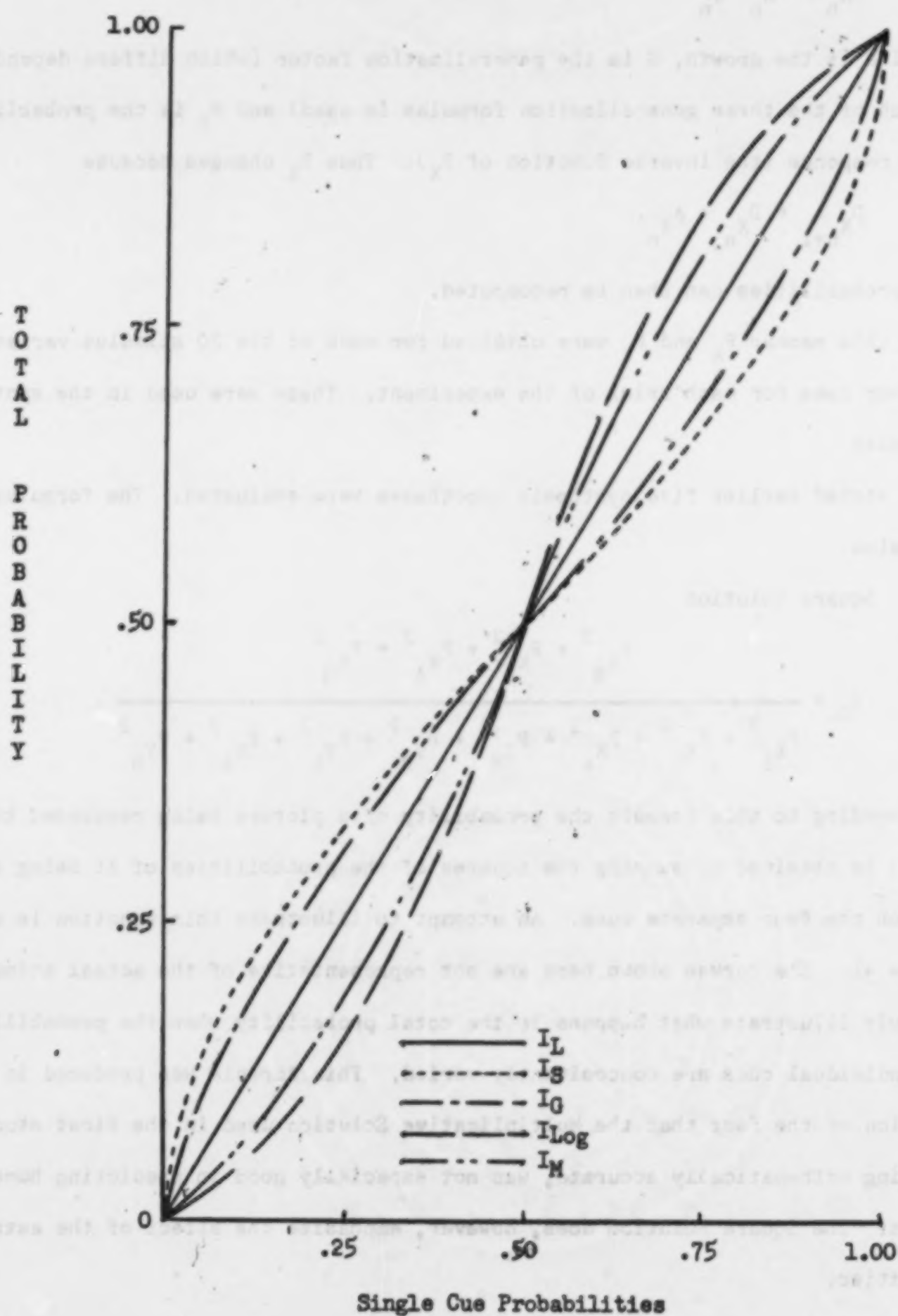
As stated earlier five synthesis hypotheses were evaluated. The formulas appear below.

1. Square Solution

$$I_S = \frac{P_{X_B}^2 + P_{X_L}^2 + P_{X_A}^2 + P_{X_H}^2}{P_{X_B}^2 + P_{X_L}^2 + P_{X_A}^2 + P_{X_H}^2 + P_{Y_B}^2 + P_{Y_L}^2 + P_{Y_A}^2 + P_{Y_H}^2}$$

According to this formula the probability of a picture being responded to as an X (I_S) is obtained by summing the squares of the probabilities of it being an X based upon the four separate cues. An attempt to illustrate this function is seen in Figure 41. The curves shown here are not representative of the actual stimuli. They simply illustrate what happens to the total probability when the probabilities of the individual cues are concomitantly varied. This formula was produced in recognition of the fact that the Multiplicative Solution used in the first study, while being mathematically accurate, was not especially good in predicting human judgments. The Square Solution does, however, emphasize the effect of the extreme probabilities.

Figure 41. Probabilities calculated for the five synthesis solutions on the basis of a concomitant change of each cue probability.



2. Geometric Solution

$$I_G = \frac{\sqrt{P_{XB} P_{XL} P_{XA} P_{XH}}}{\sqrt{P_{XB} P_{YL} P_{XA} P_{XH}} + \sqrt{P_{YB} P_{YL} P_{YA} P_{YH}}}$$

This formula is similar to the Square Solution in that it gives additional emphasis to the extreme probability values.

3. Linear Solution

$$I_L = \frac{P_{XB} + P_{XL} + P_{XA} + P_{XH}}{P_{XB} + P_{XL} + P_{XA} + P_{XH} + P_{YB} + P_{YL} + P_{YA} + P_{YH}}$$

This formula proved the most accurate predictor in Studies 1 and 2.

4. Log Solution

$$I_{Log} = \frac{\log_{10} (P_{XB} P_{XL} P_{XA} P_{XH})}{\log_{10} (P_{XB} P_{XL} P_{XA} P_{XH} P_{YB} P_{YL} P_{YA} P_{YH})}$$

This formula gives disproportionate emphasis to the middle range of probability values. Mathematically it is the least precise of the hypotheses evaluated. It was included since there was evidence in Study No. 1 that with normal subjects it tended to improve as a predictor as testing continued.

5. Modified Log Solution

$$I_M = \frac{I_{Log} + I_L}{2}$$

As can be seen this formula is not so extreme as I_{Log} . It was included as a check on the range between the Log and Linear Solutions.

All these functions are illustrated in Figure 41. Again it should be remembered that these curves apply only to the situation where there is a gradual and concomitant increase in the probabilities of the individual cues.

Each of the above solutions for stimulus synthesis was applied in combination with each of the solutions for generalization against the empirical data by means of correlations.

Correlations were chosen in evaluation of the data rather than error scores because they automatically adjust themselves to the slope of the function. They have the disadvantage of being insensitive particularly as the models become more accurate but are useful enough for the preliminary work that this research exemplifies.

The results of this study are shown in Figures 42 through 80. Since the correlations are generally high (.80 to .97 in the fourth block of trials) the differences are not always great. Consequently, the consistency of the differences rather than their magnitude will be emphasized.

In Figures 56 through 80 comparison is made of the three subject populations for the fifteen (5 synthesis formulas x 3 generalization formulas) models. It is apparent from these graphs that all of the models are a better description of retarded than normal behavior, during the last 300 trials of the experiment. It is also true that better predictions were obtained on the HS group than on the GS group on the first, second, and fourth blocks of trials. Each of these statements are based upon the results of a Frieden test (Siegel, 1956) showing a .01 probability of occurrence.

In Figures 57 through 71 comparison is obtained of the generalization hypotheses. For the normals the G_H is consistently the better predictor. This is followed by G_L and finally G_S . These differences are also significant at the 99% level of confidence.

Figure 42. Correlations of the High School, Grade School, and Retarded Groups for Linear Generalization (G_L) and Linear Integration (I_L).

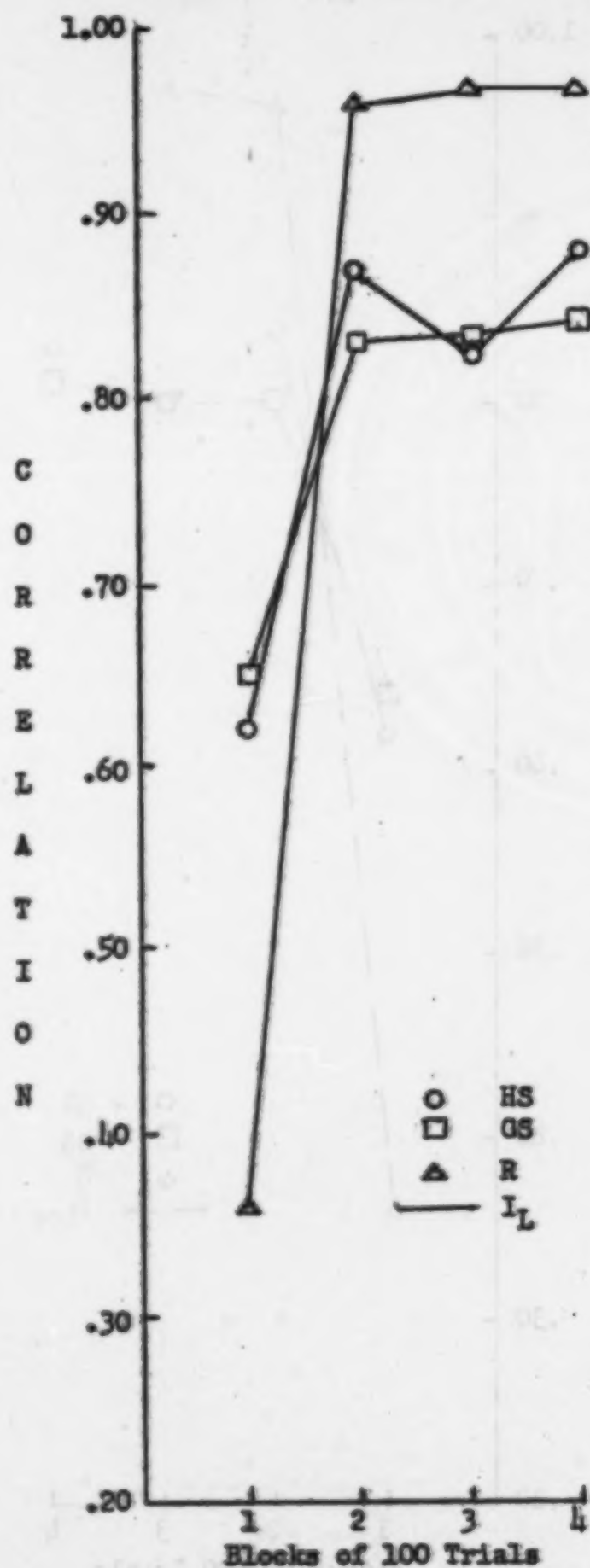


Figure 43. Correlations of the High School, Grade School, and Retarded Groups for Linear Generalization (G_L) and Squared Integration (I_S).

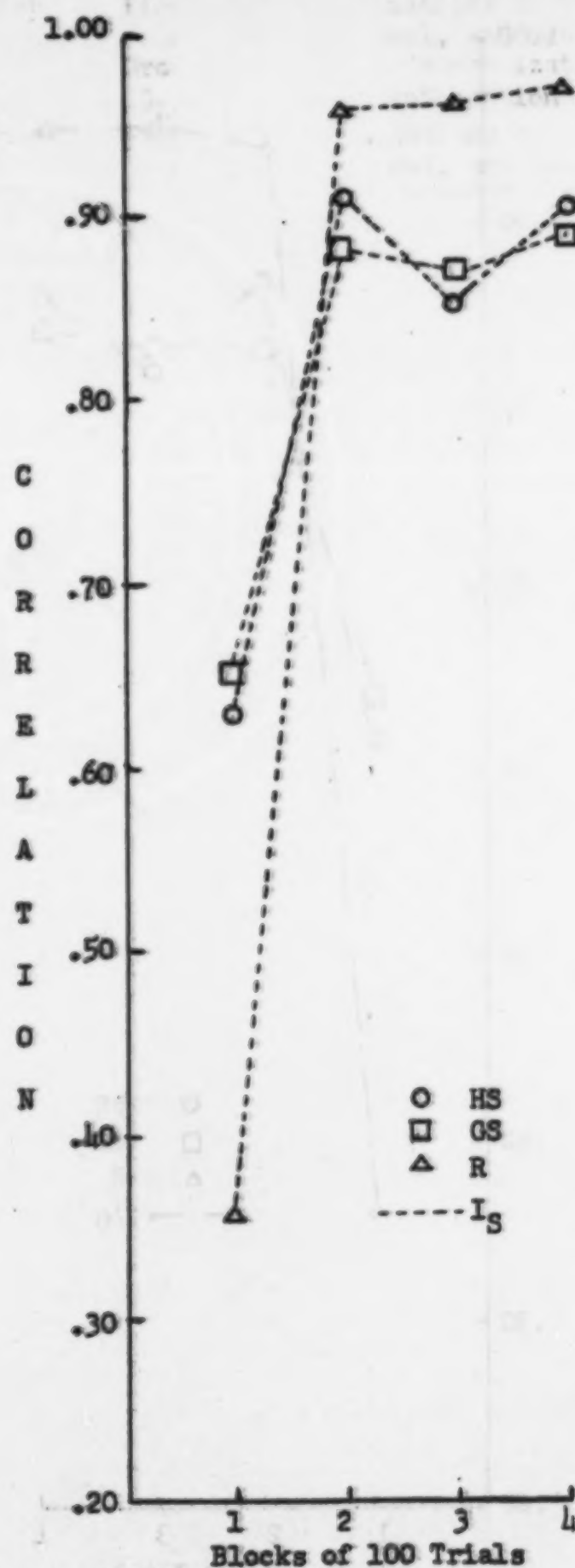


Figure 44. Correlations of the High School, Grade School, and Retarded Groups for Linear Generalization (G_L) and Geometric Integration (I_G).

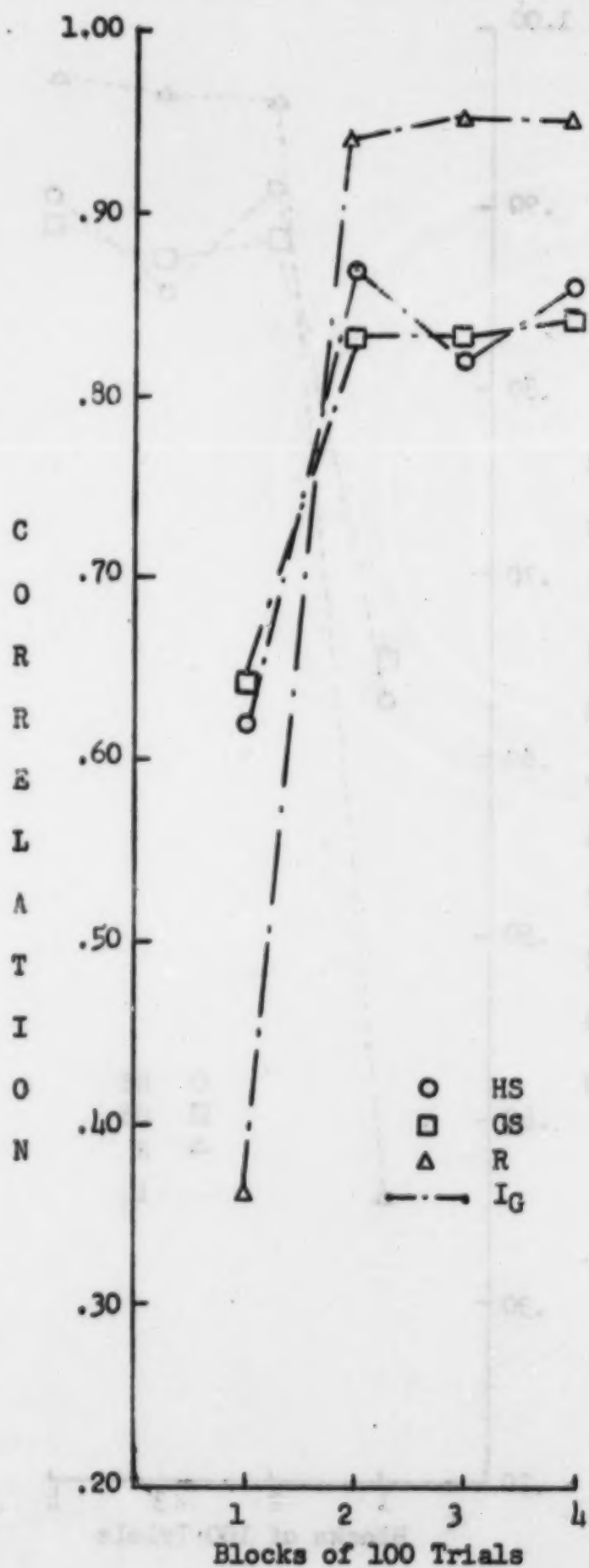


Figure 45. Correlations of the High School, Grade School, and Retarded Groups for Linear Generalization (G_L) and Logarithmic Integration (I_{Log}).

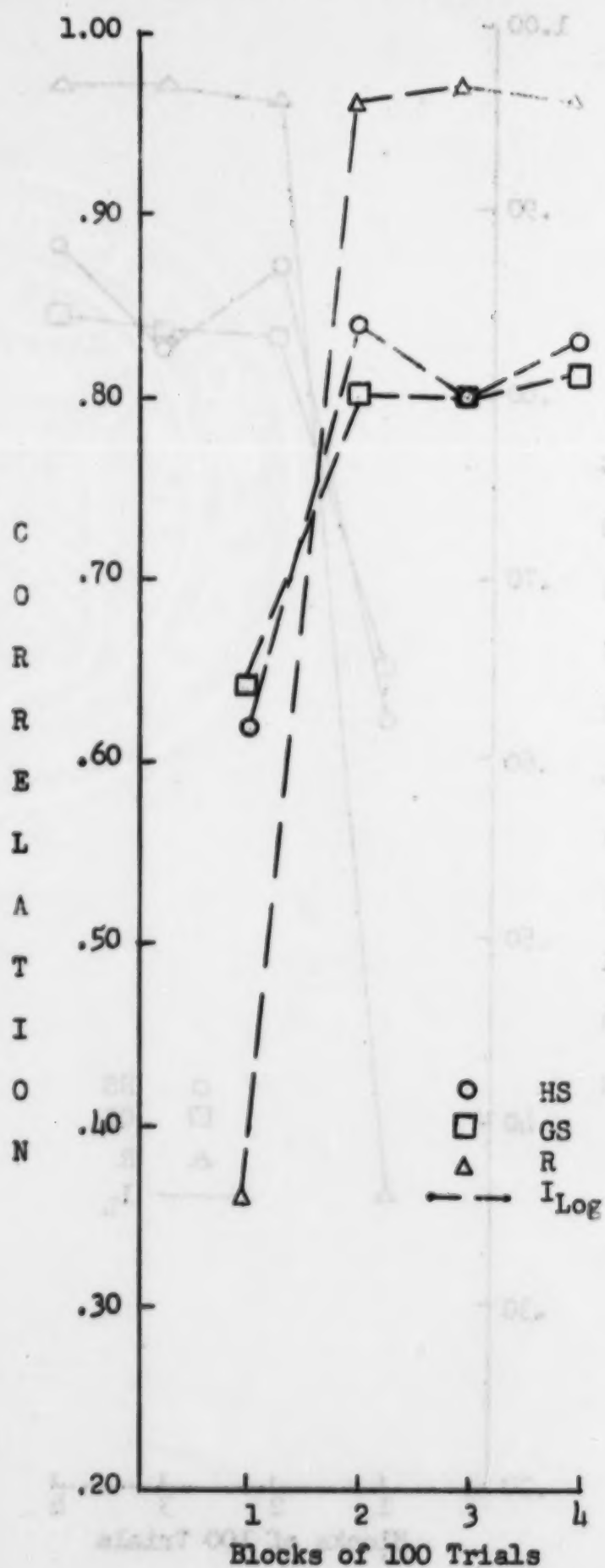


Figure 46. Correlations of the High School, Grade School, and Retarded Groups for Linear Generalization (G_L) and Modified Log Integration (I_M).

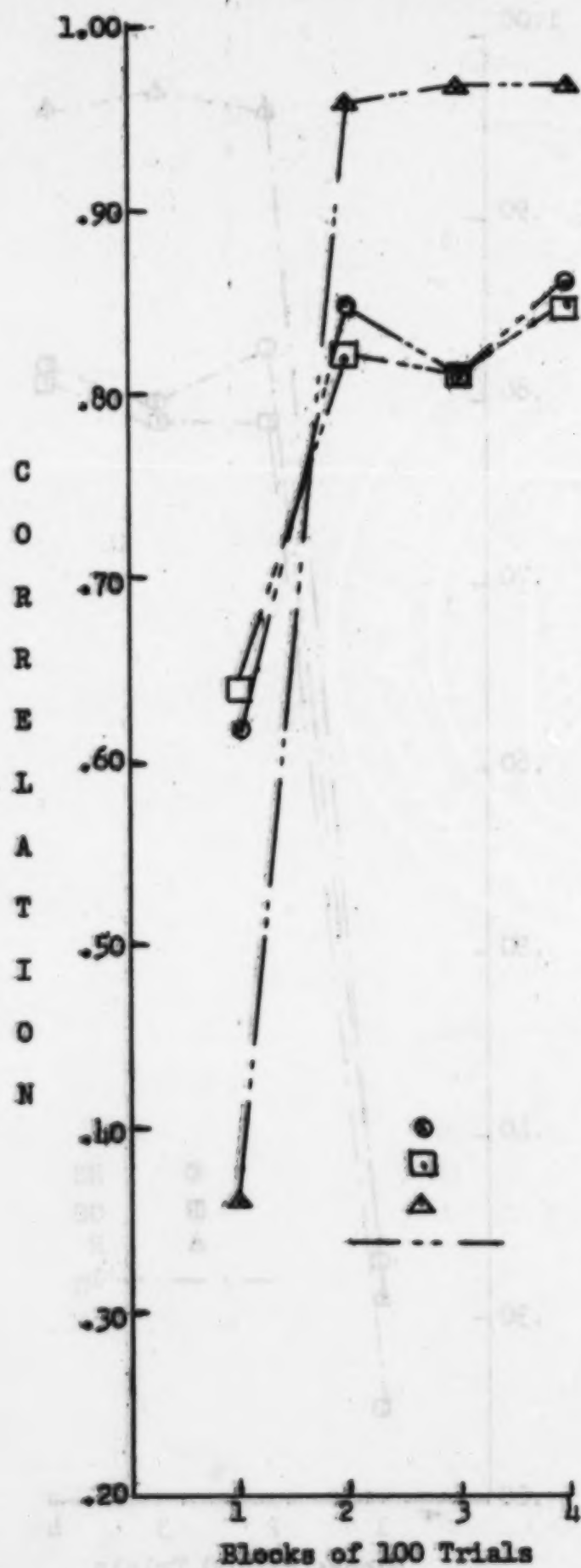


Figure 47. Correlations of the High School, Grade School, and Retarded Groups for Generalization S (G_S) and Linear Integration (I_L).

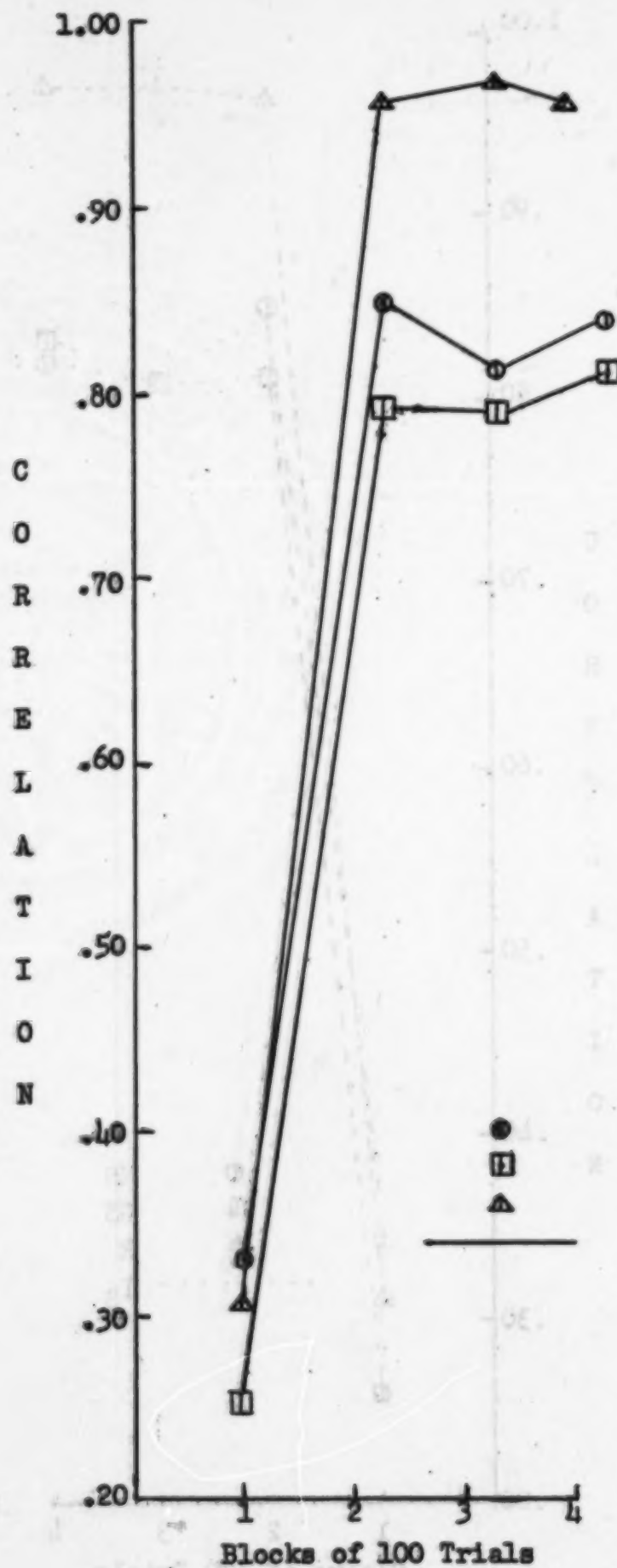


Figure 48. Correlations of the High School, Grade School, and Retarded Groups for Generalization S (G_S) and Squared Integration (I_S).

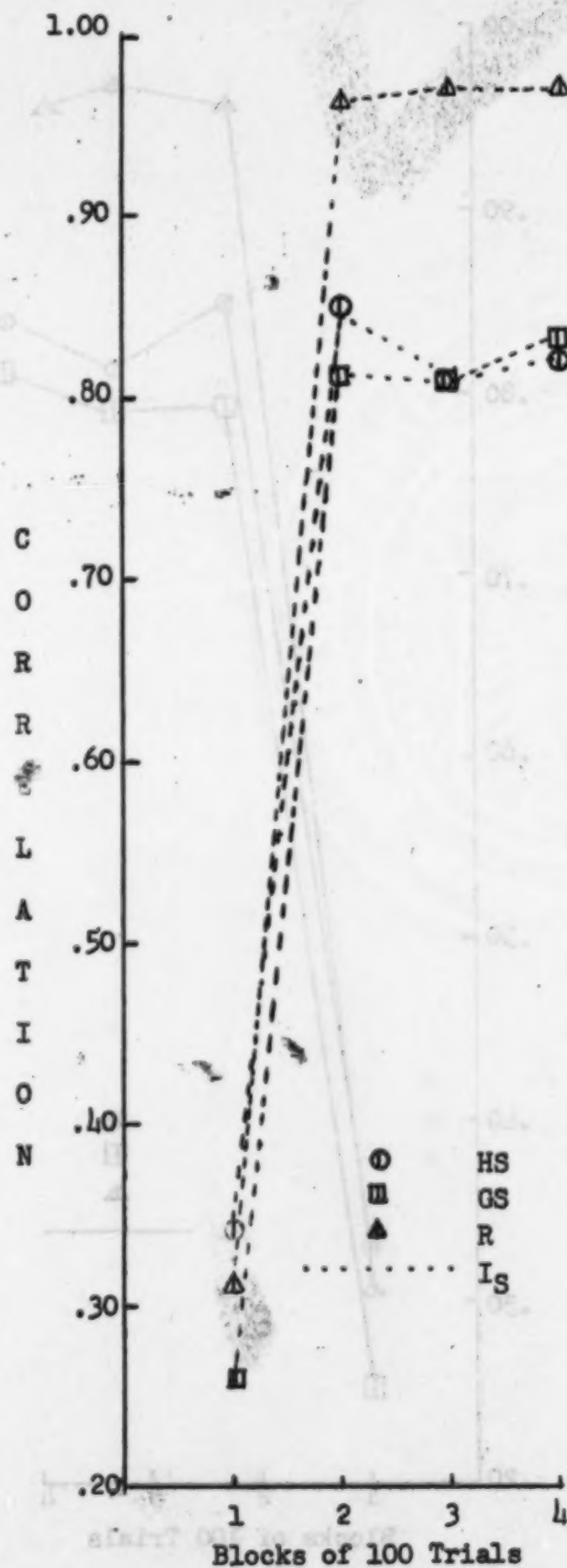


Figure 49. Correlations of the High School, Grade School, and Retarded Groups for Generalization S (G_S) and Geometric Integration (I_G).

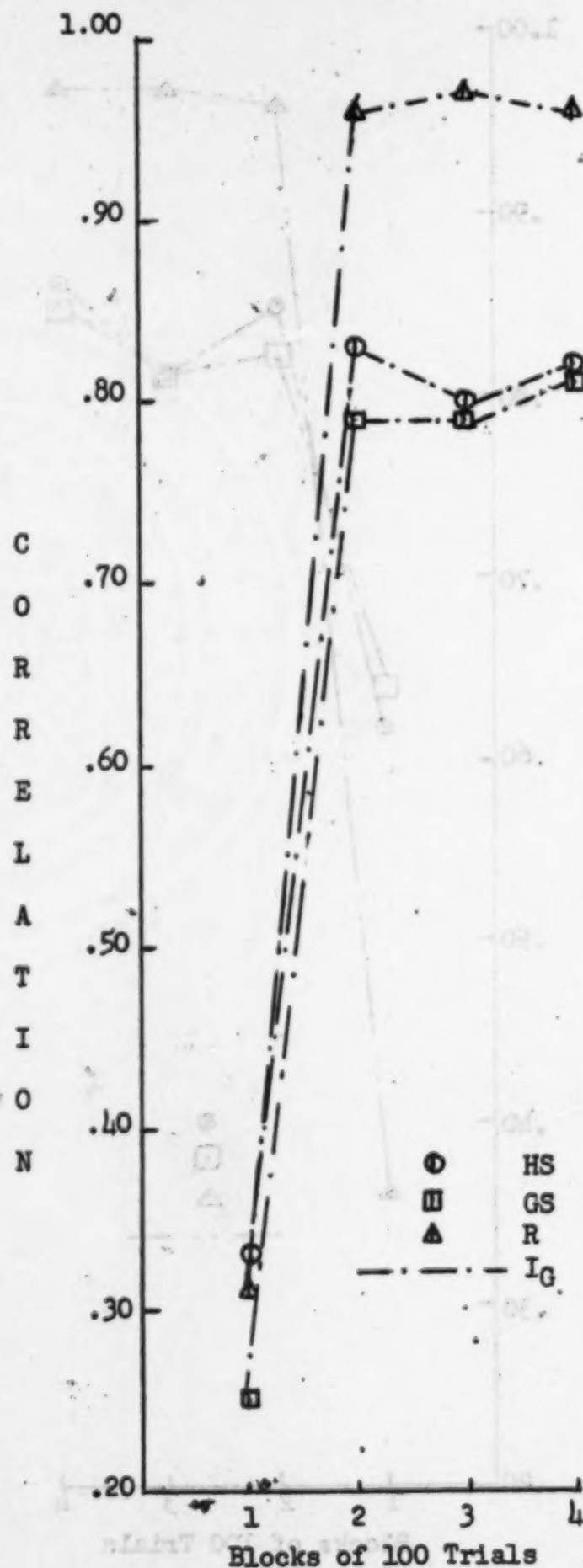


Figure 50. Correlations of the High School, Grade School, and Retarded Groups for Generalization S (G_S) and Logarithmic Integration (I_{Log}).

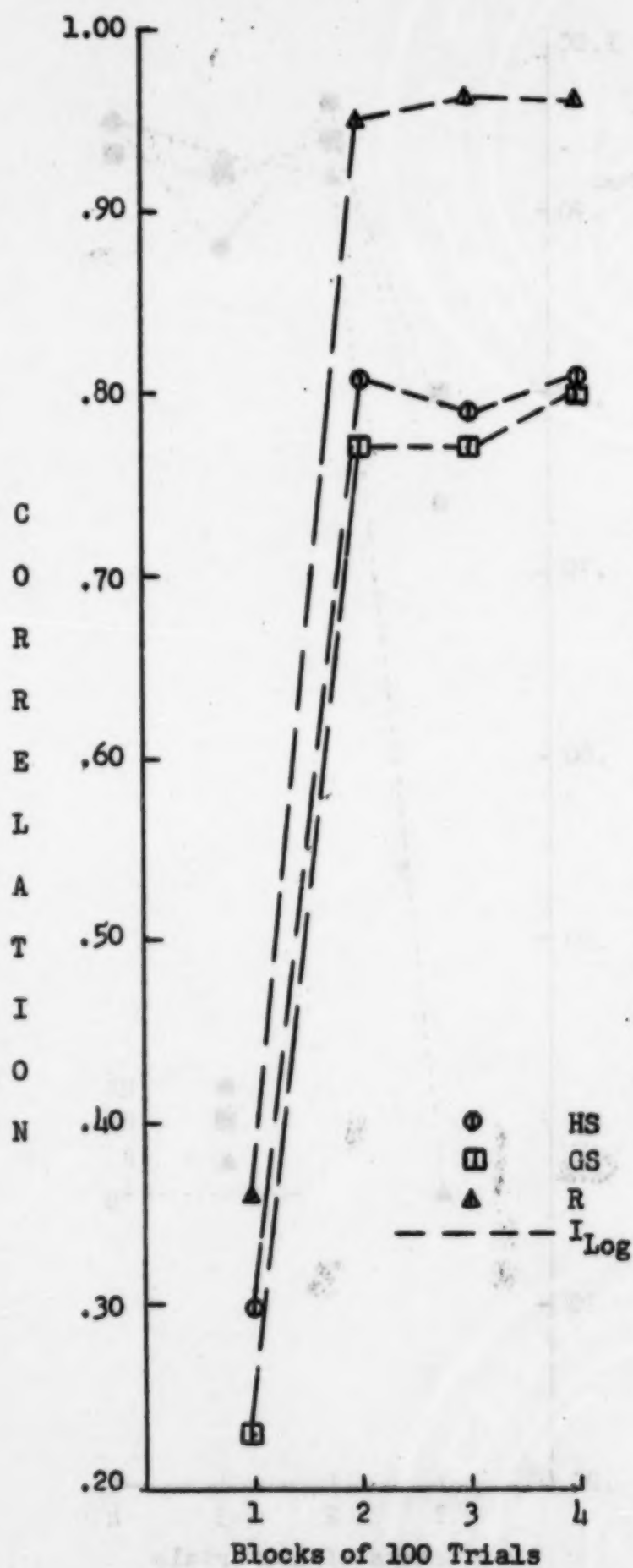


Figure 51. Correlations of the High School, Grade School, and Retarded Groups for Generalization S (G_S) and Modified Log Integration (I_M).

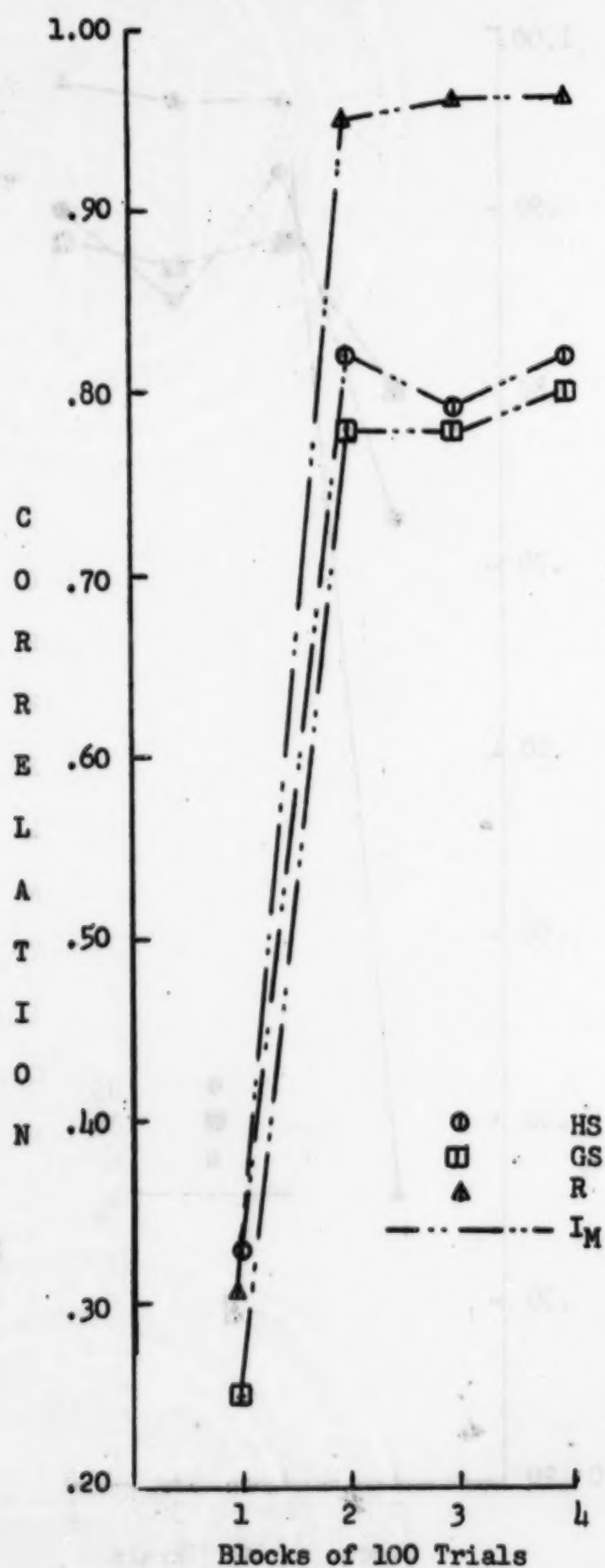


Figure 52. Correlations of the High School, Grade School, and Retarded Groups for the Generalization H (G_H) and Linear Integration (I_L).

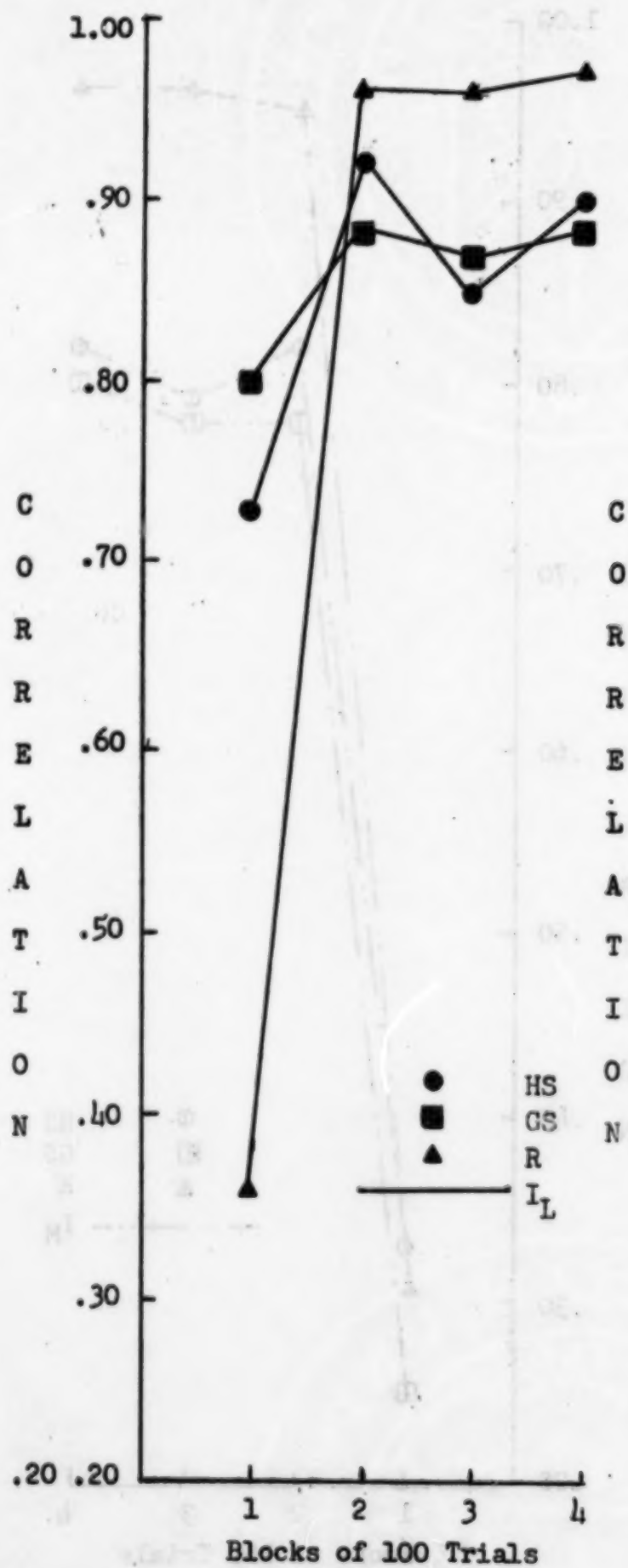


Figure 53. Correlations of the High School, Grade School, and Retarded Groups for the Generalization H (G_H) and Squared Integration (I_S).

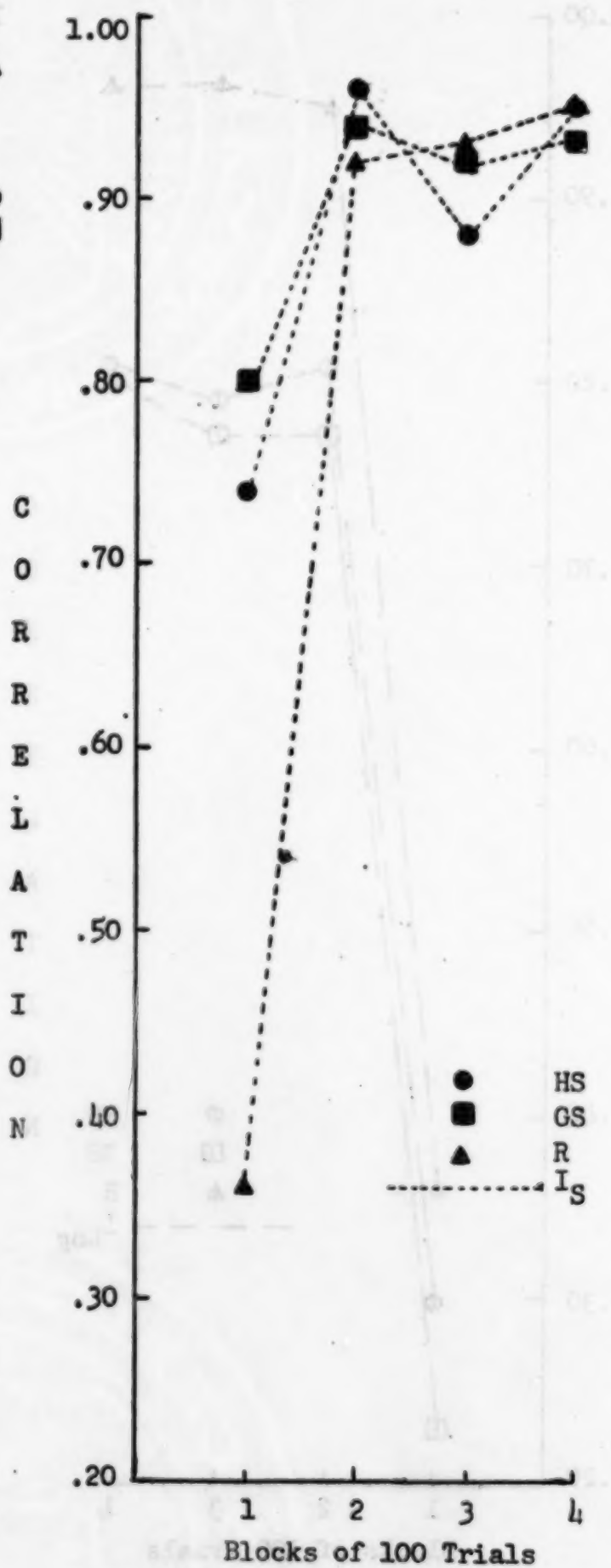


Figure 54. Correlations of the High School, Grade School, and Retarded Groups for the Generalization H (G_H) and Geometric Integration (I_G).

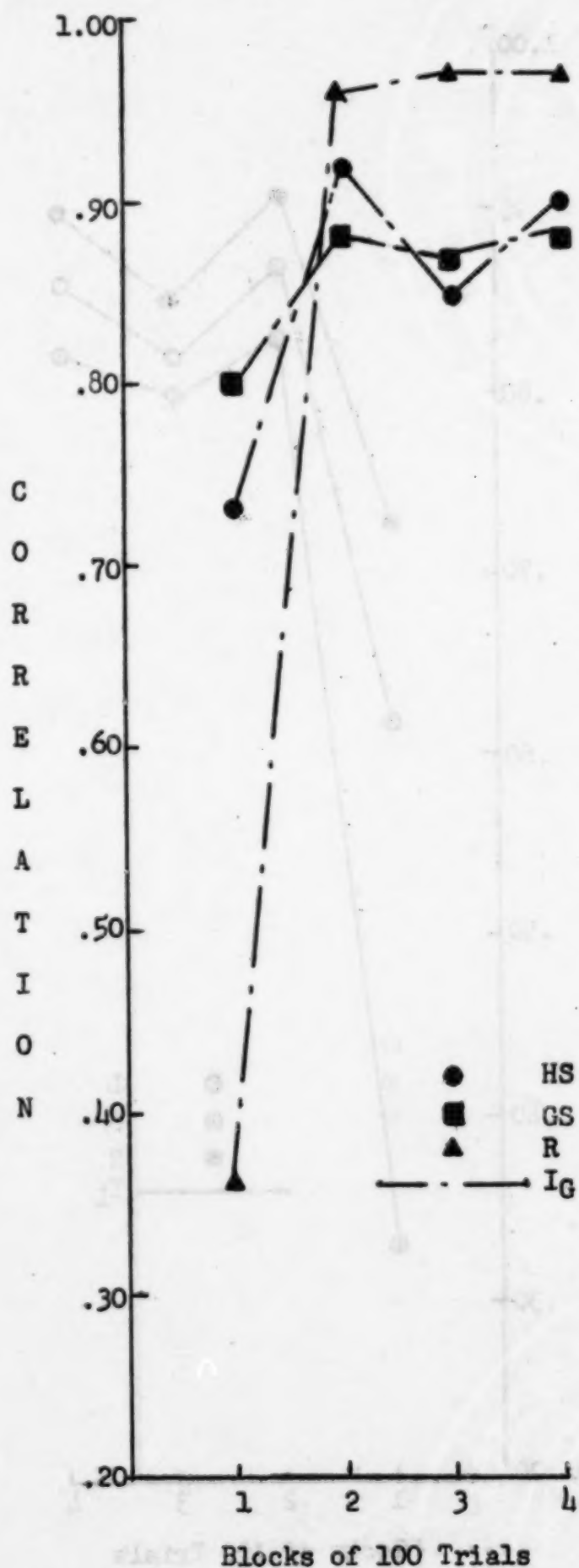


Figure 55. Correlations of the High School, Grade School, and Retarded Groups for the Generalization H (G_H) and Logarithmic Integration (I_{Log}).

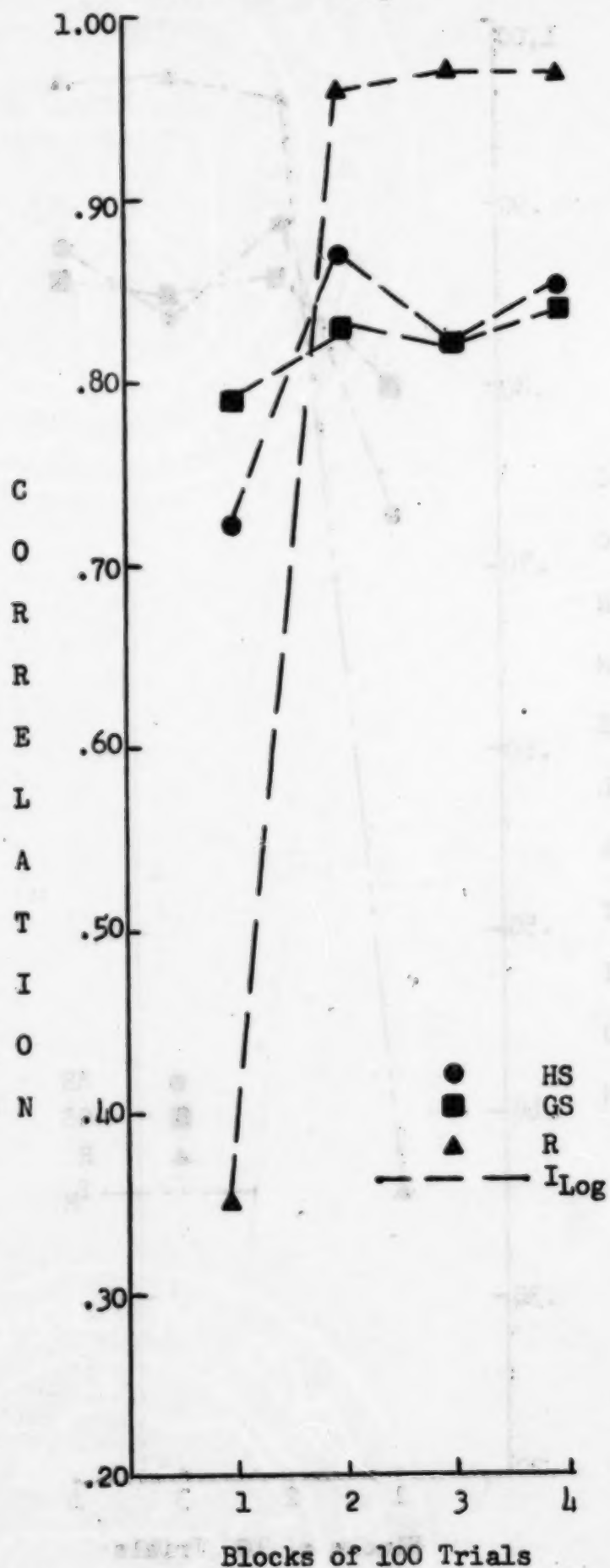


Figure 56. Correlations of the High School, Grade School, and Retarded Groups for the Generalization H (G_H) and Modified Log Integration (I_M).

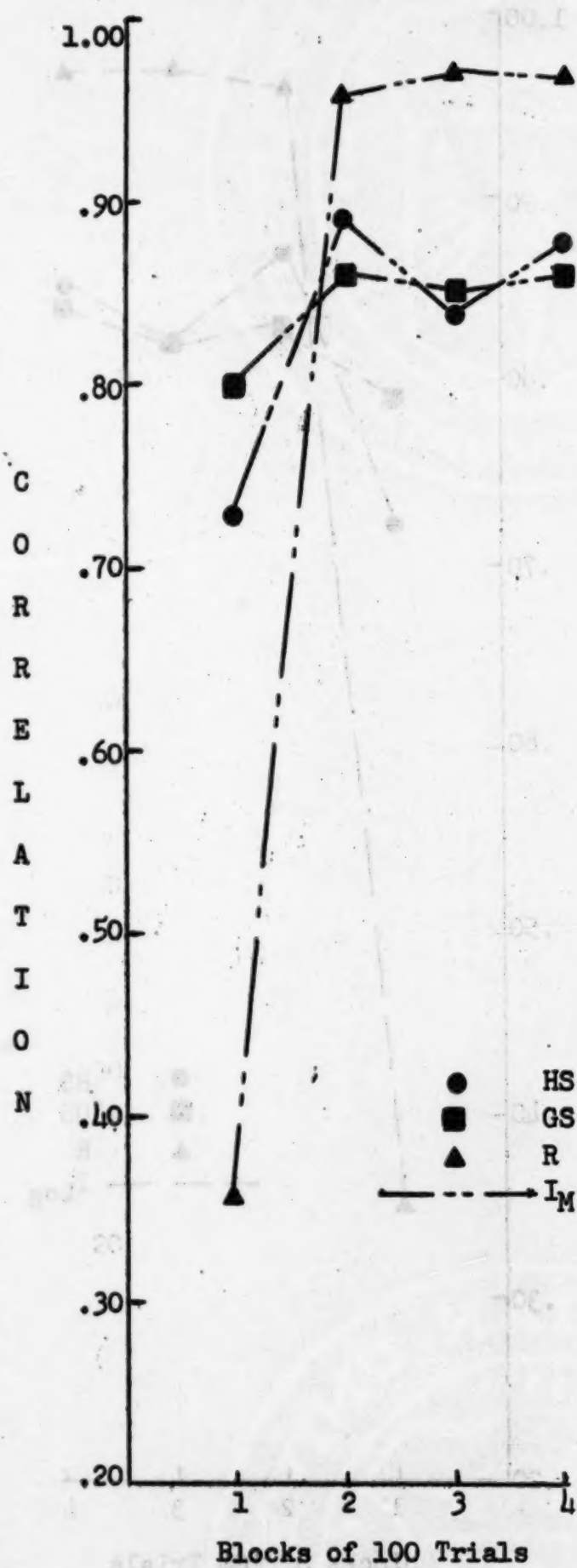


Figure 57. Correlations of Generalizations L, S, and H with Linear Integration for the High School Group.

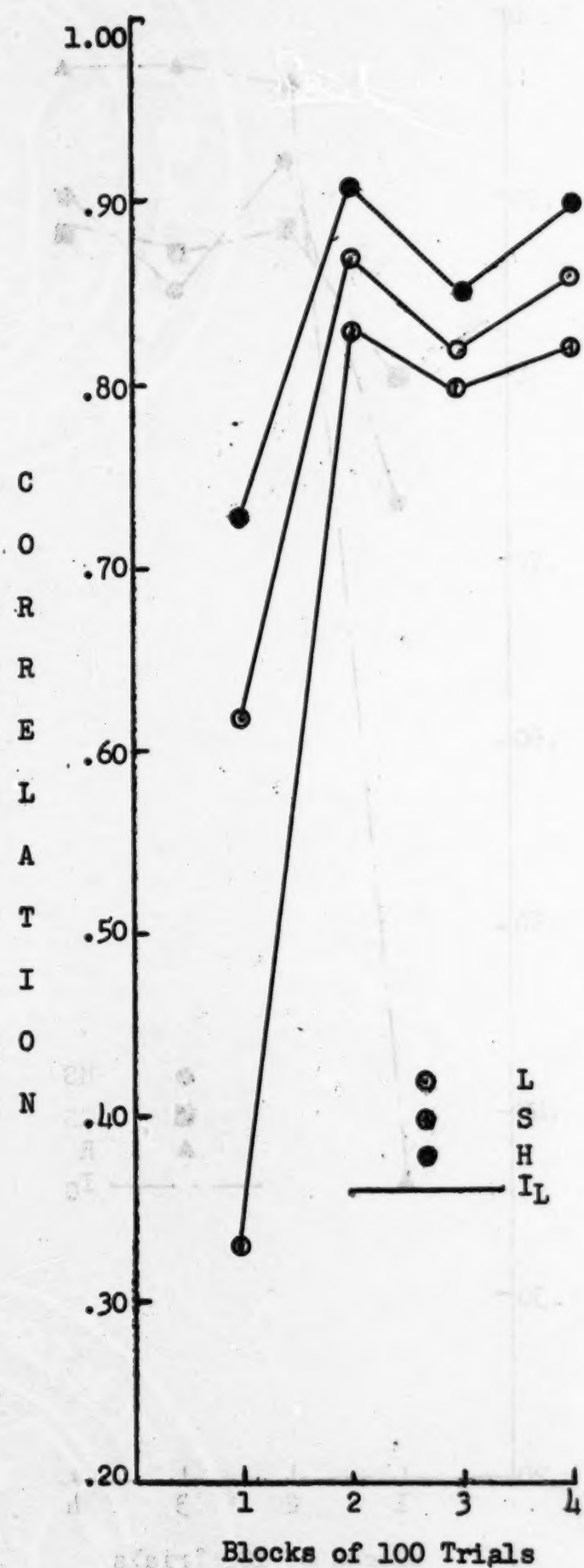


Figure 58. Correlations of Generalizations L, S, and H with Squared Integration for the High School Group.

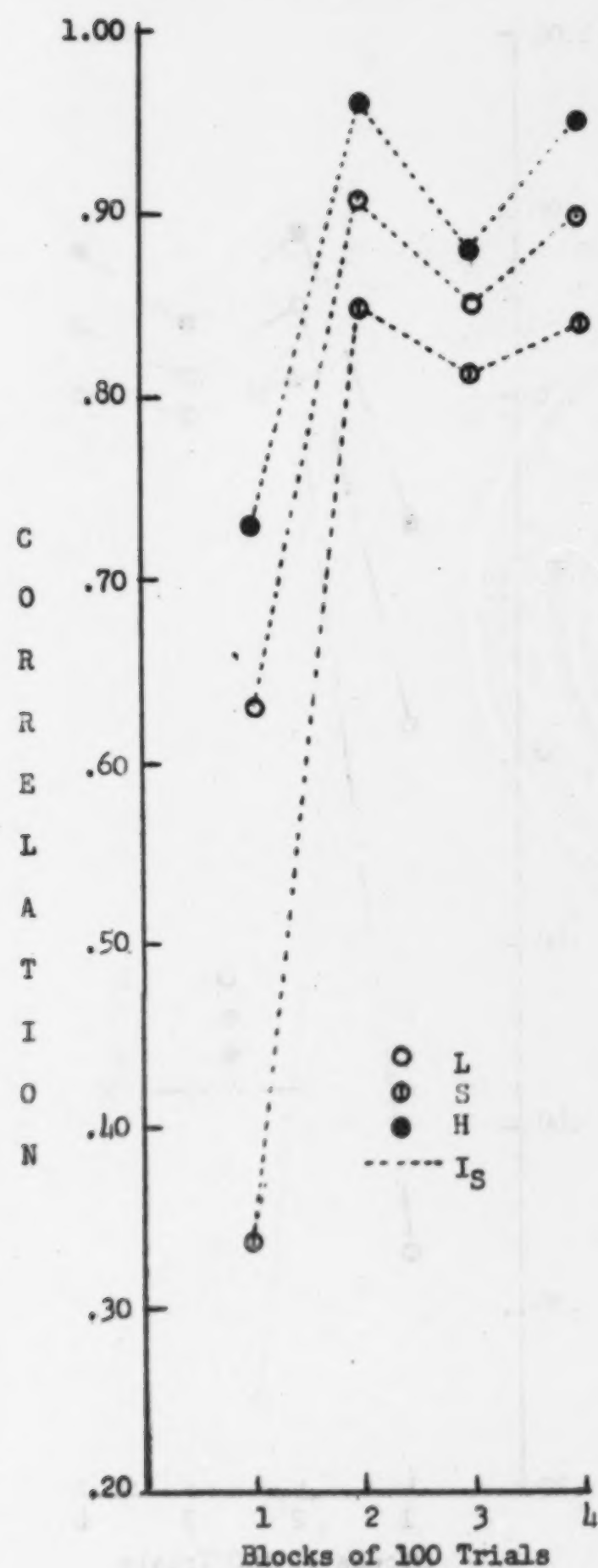


Figure 59. Correlations of Generalizations L, S, and H with Geometric Integration for the High School Group.

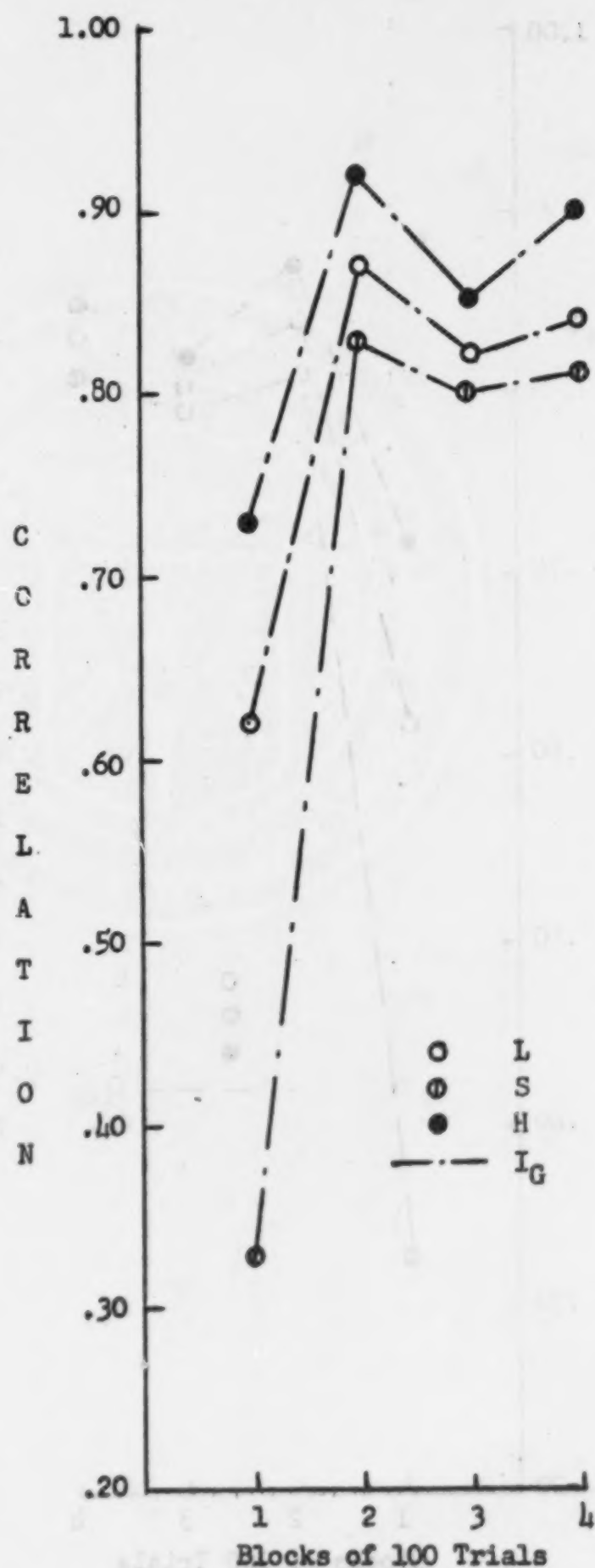


Figure 60. Correlations of Generalizations L, S, and H with Logarithmic Integration for the High School Group.

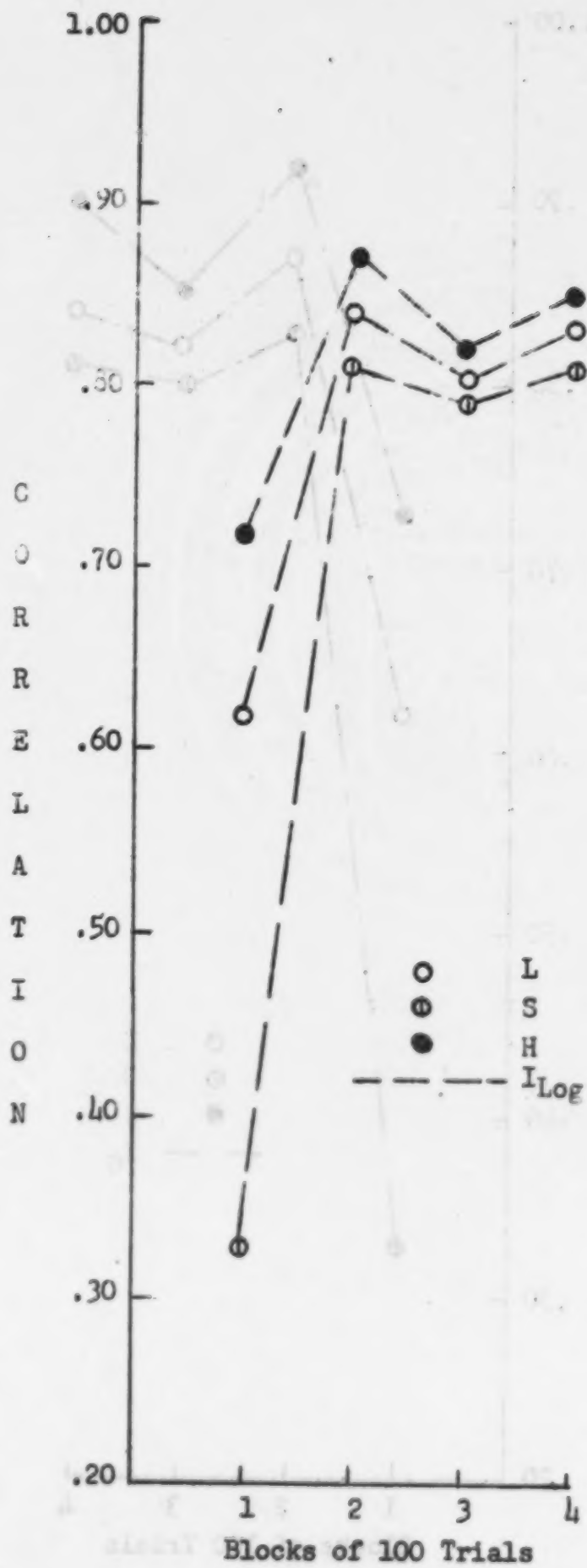


Figure 61. Correlations of Generalizations L, S, and H with Modified Log Integration for the High School Group.

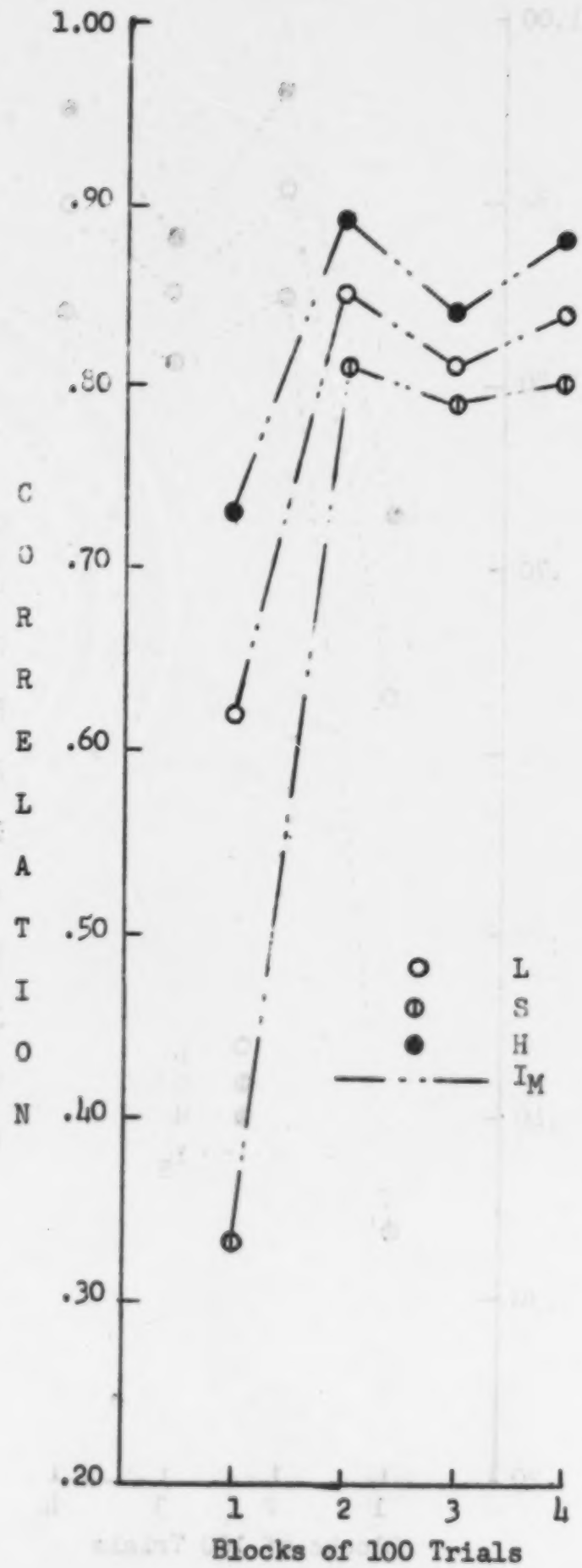


Figure 62. Correlations of Generalizations L, S, and H with Linear Integration for the Grade School Group.

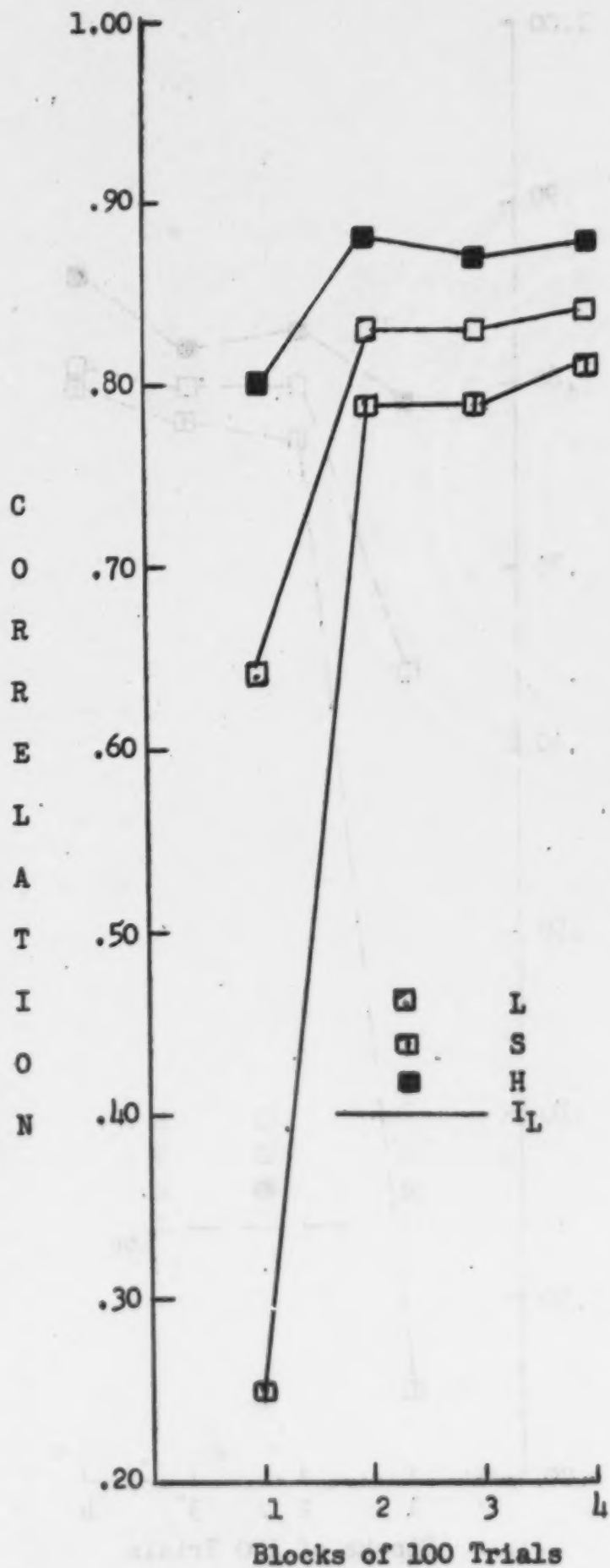


Figure 63. Correlations of Generalizations L, S, and H with Squared Integration for the Grade School Group.

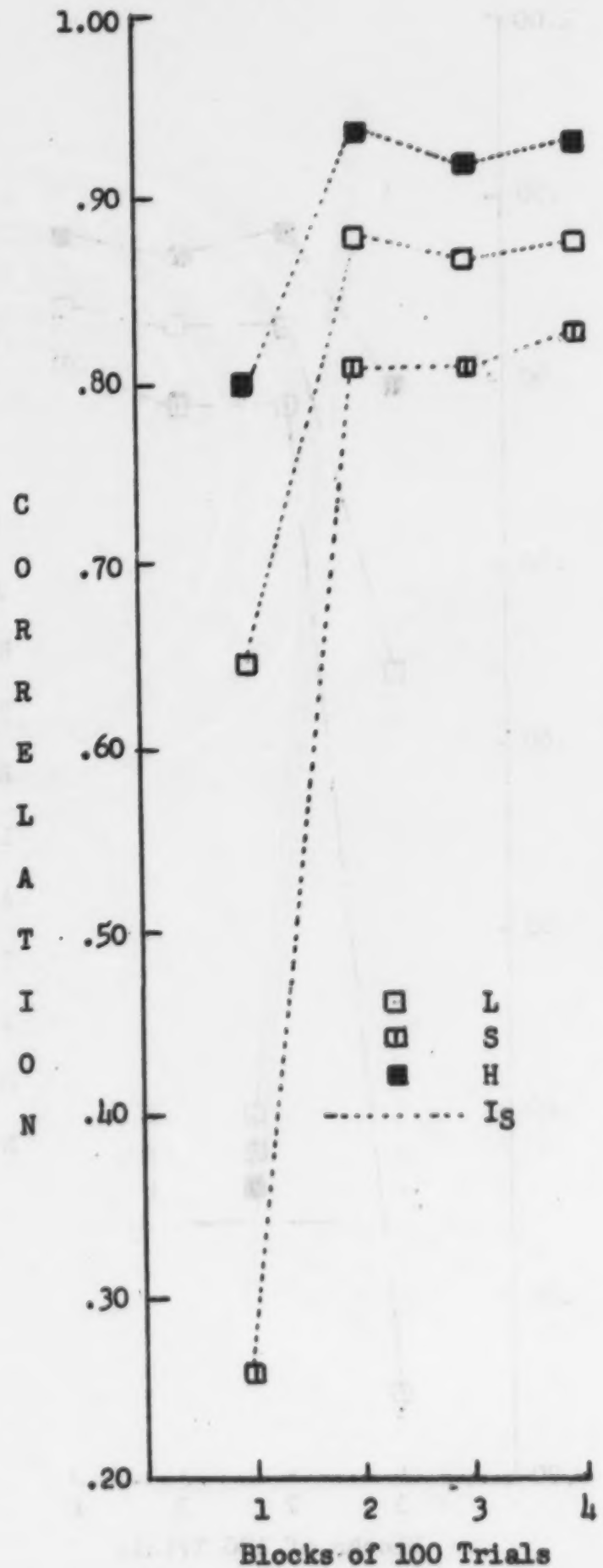


Figure 64. Correlations of Generalizations L, S, and H with Geometric Integration for the Grade School Group.

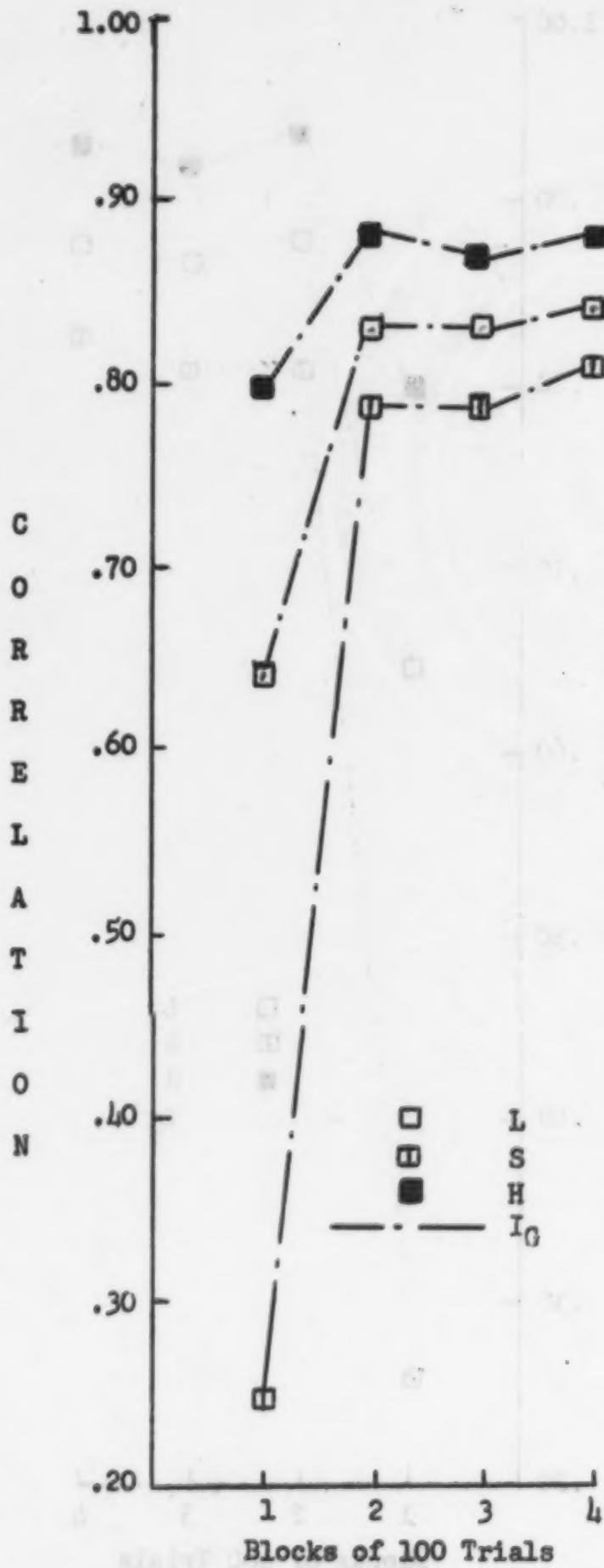


Figure 65. Correlations of Generalizations L, S, and H with Logarithmic Integration for the Grade School Group.

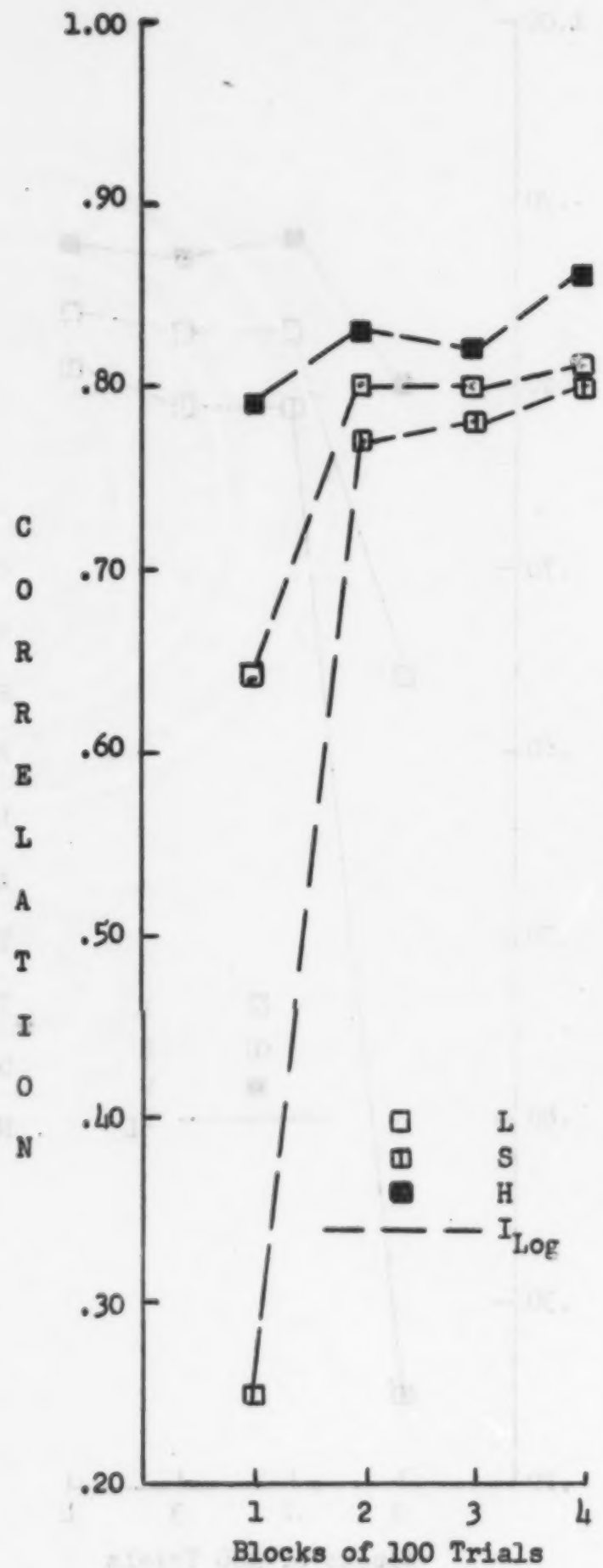


Figure 66. Correlations of Generalizations L, S, and H with Modified Log Integration for the Grade School Group.

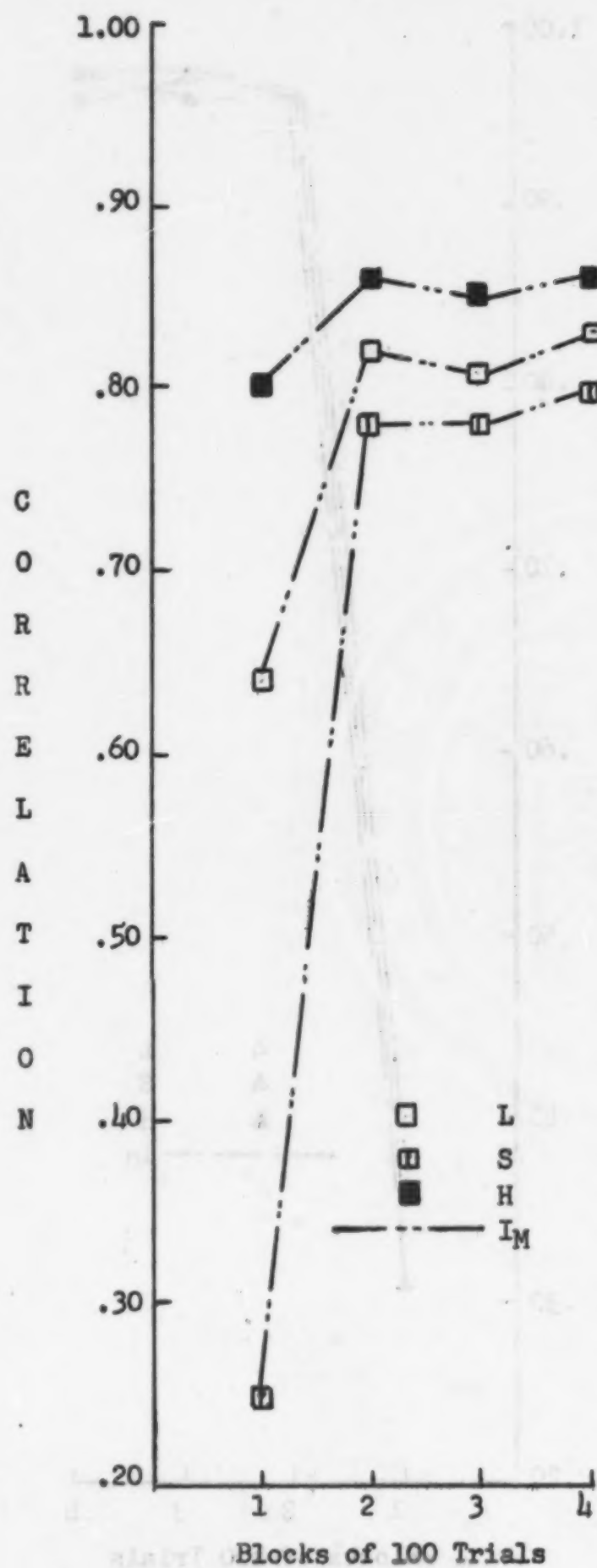


Figure 67. Correlations of Generalizations L, S, and H with Linear Integration for the Retarded Group.

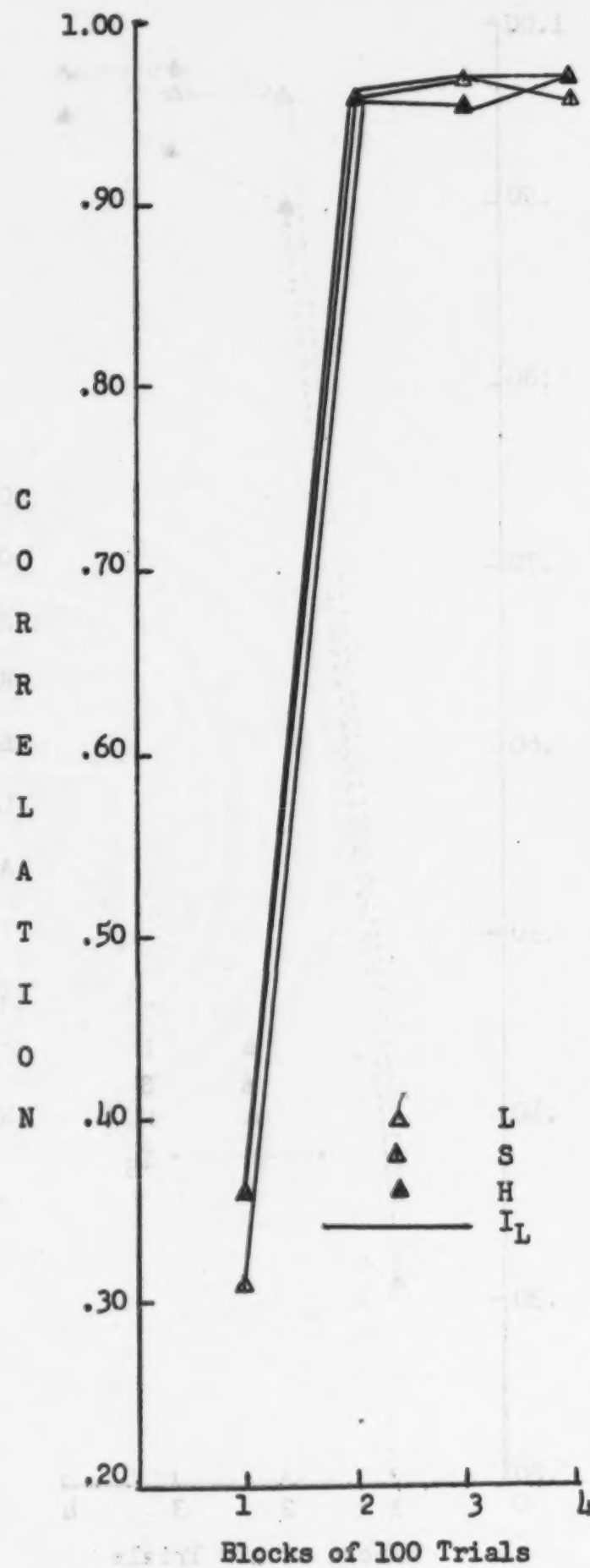


Figure 68. Correlations of Generalizations L, S, and H with Squared Integration for the Retarded Group.

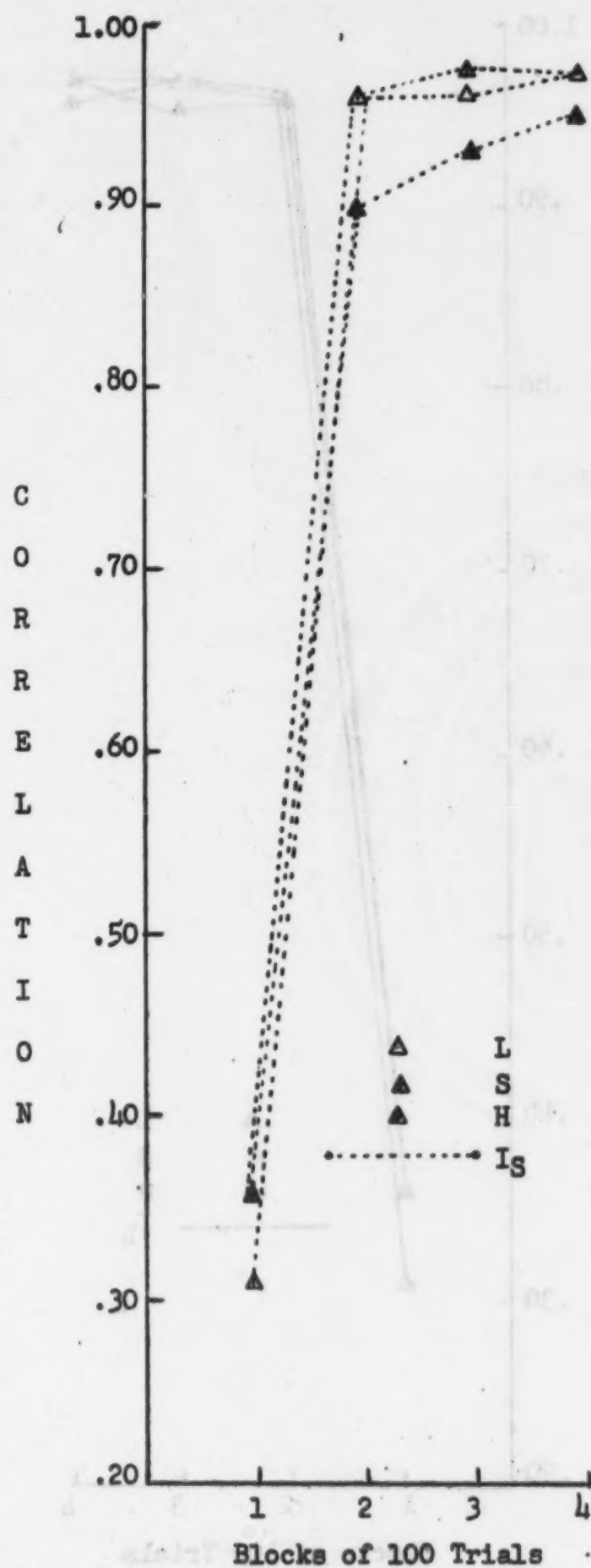


Figure 69. Correlations of Generalizations L, S, and H with Geometric Integration for the Retarded Group.

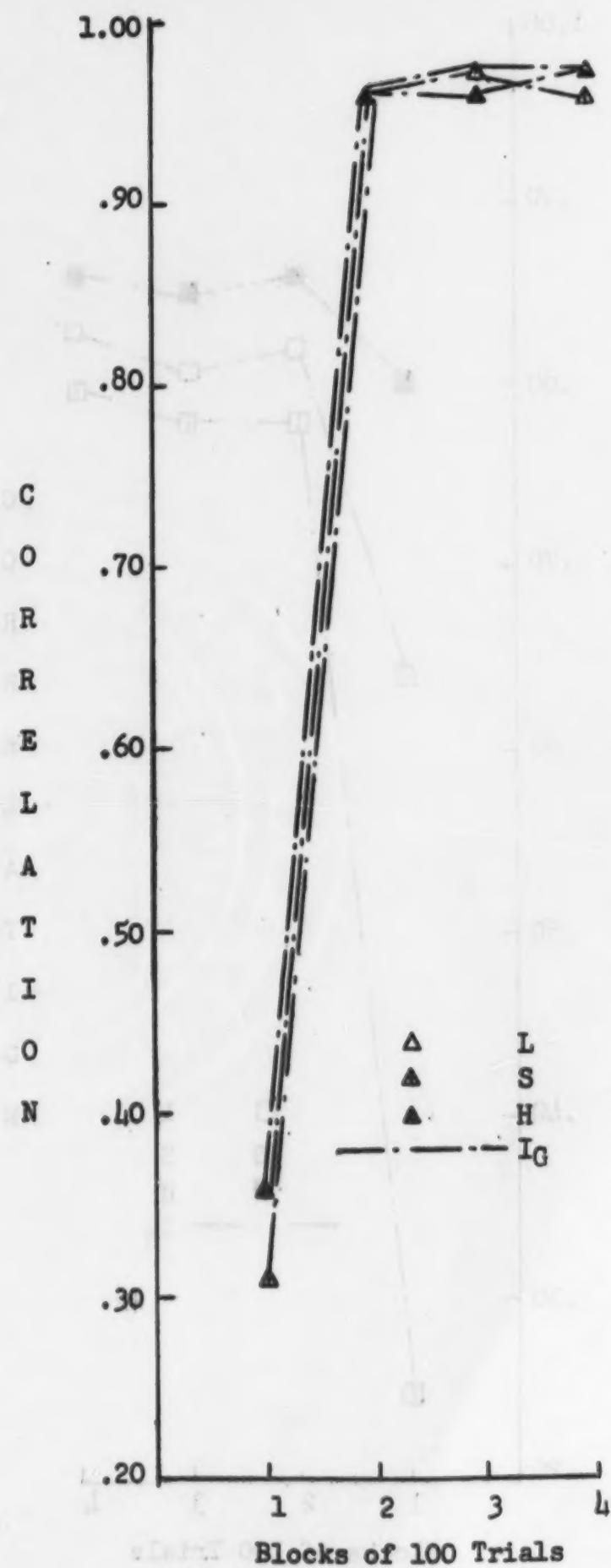


Figure 70. Correlations of Generalizations L, S, and H with Logarithmic Integration for the Retarded Group.

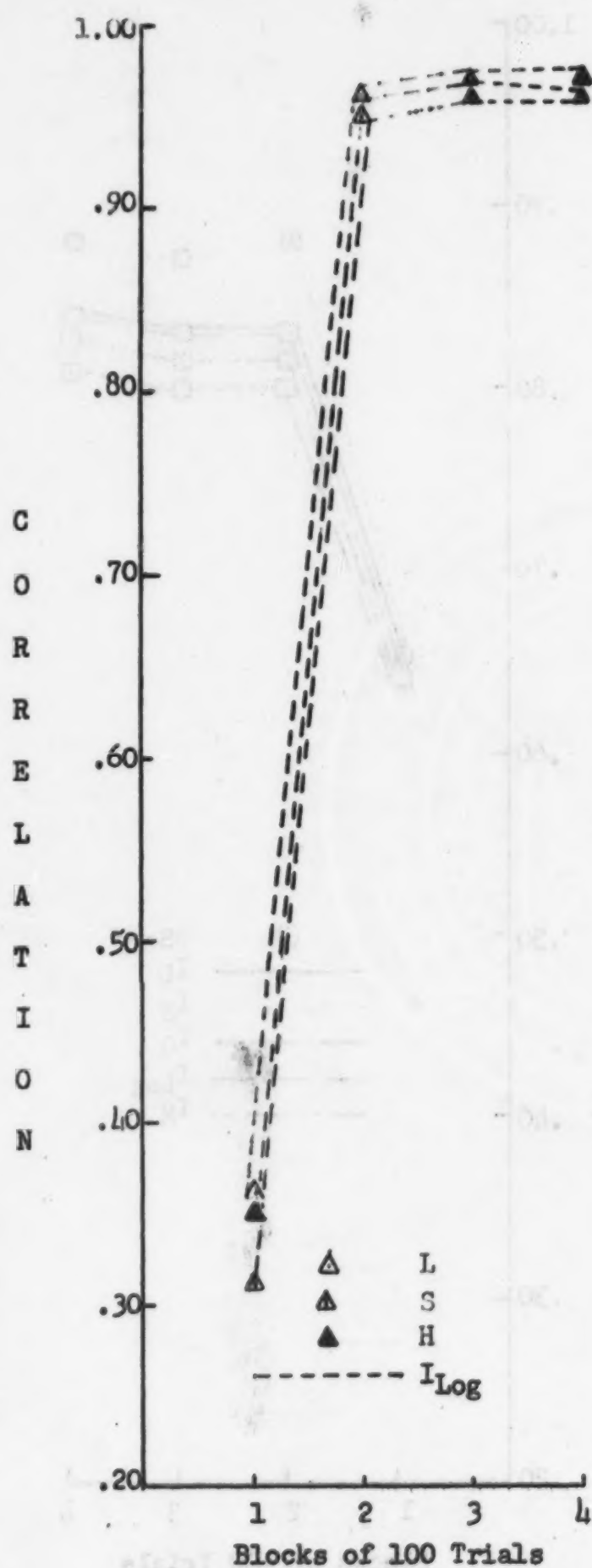


Figure 71. Correlations of Generalizations L, S, and H with Modified Log Integration for the Retarded Group.

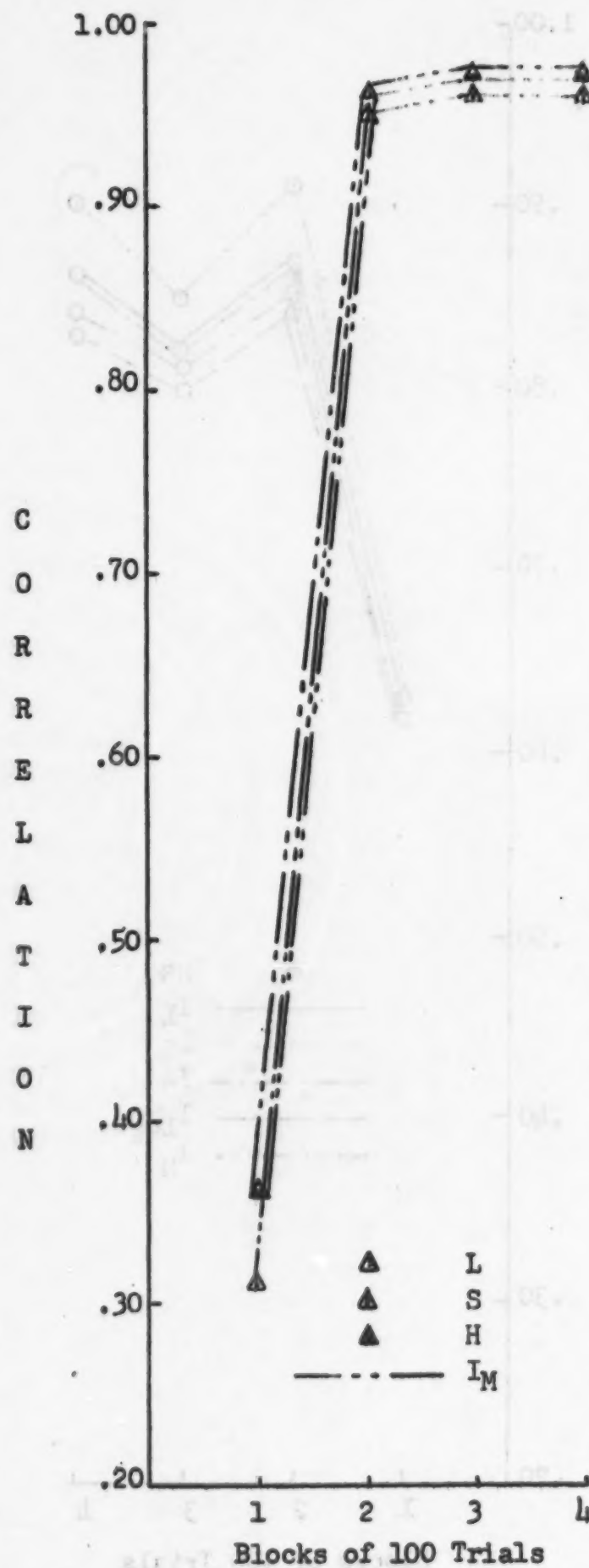


Figure 72. Correlations of the Linear Generalization for the High School Group with Linear, Squared, Geometric, Logarithmic, and Modified Log Integration methods.

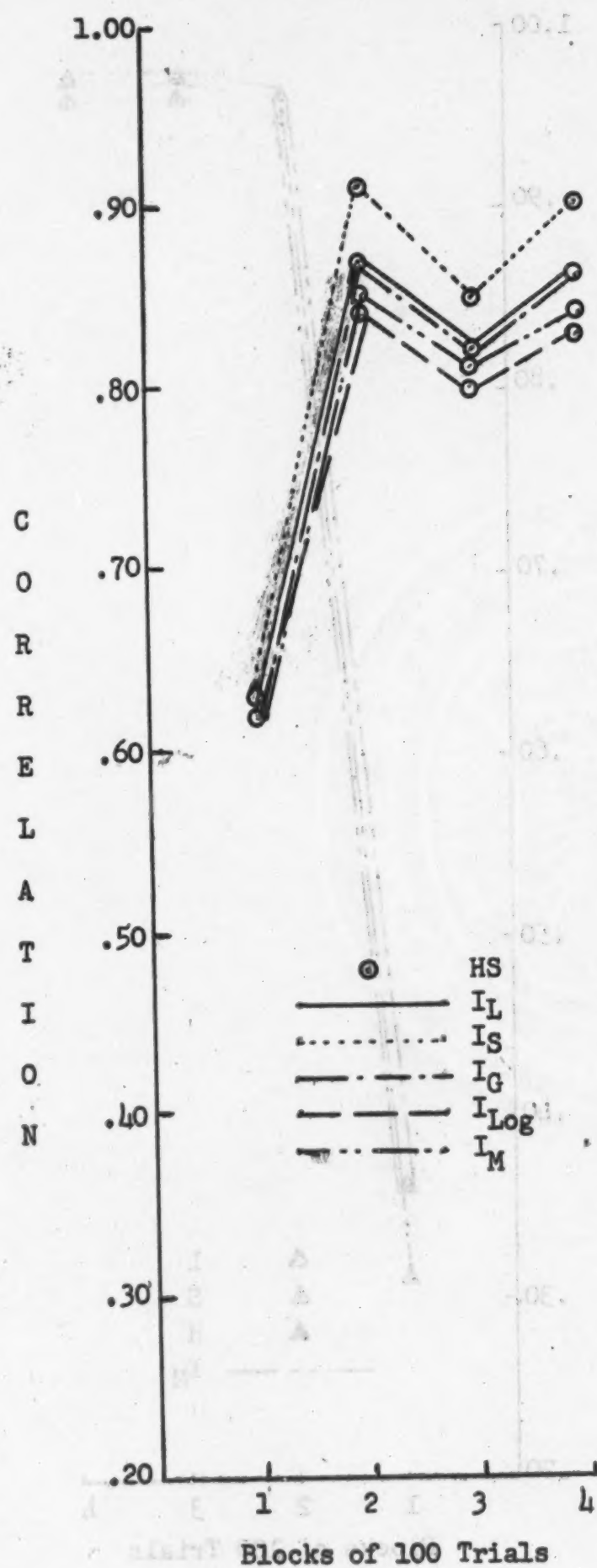
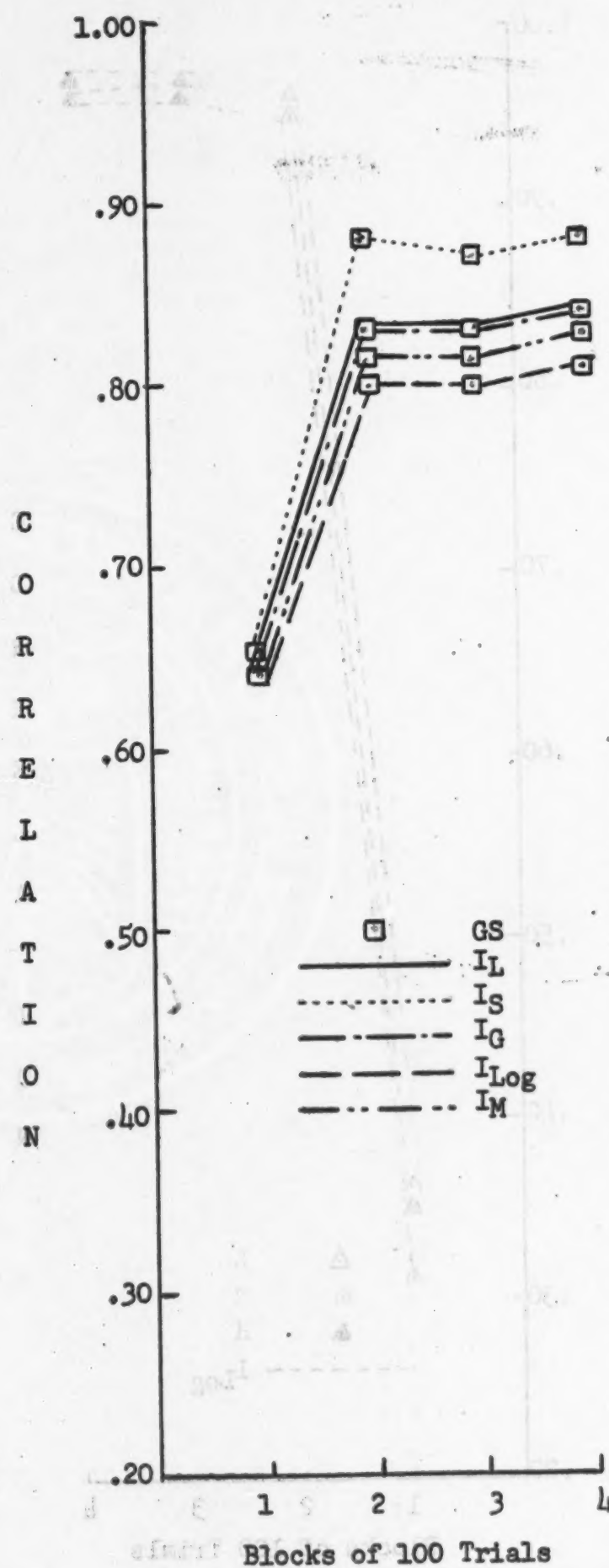


Figure 73. Correlations of the Linear Generalization for the Grade School Group with Linear, Squared, Geometric, Logarithmic, and Modified Log Integration methods.





100 to 1
 100 to 1
 100 to 1
 100 to 1
 100 to 1



Figure 77. Correlations of the Generalization S for the Retarded Group with Linear, Squared, Geometric, Logarithmic, and Modified Log Integration methods.

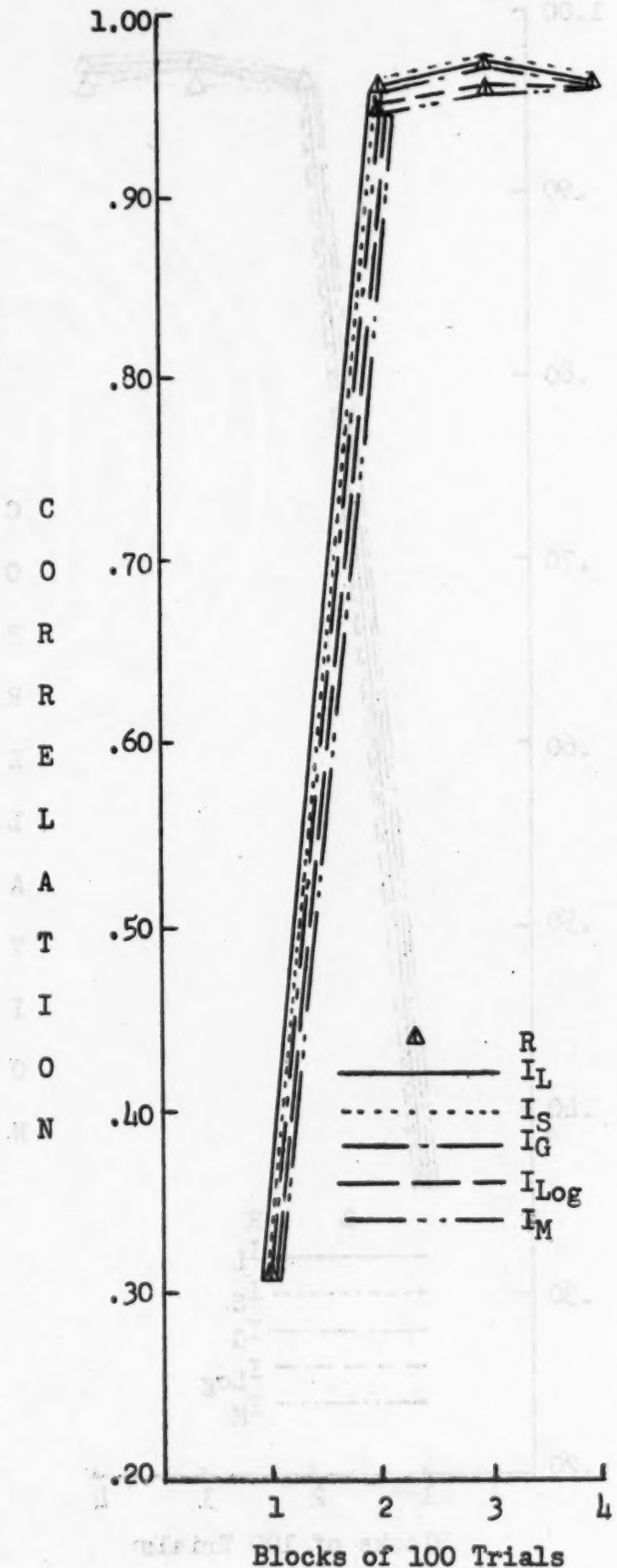


Figure 78. Correlations of the Generalization H for the High School Group with Linear, Squared, Geometric, Logarithmic, and Modified Log Integration methods.

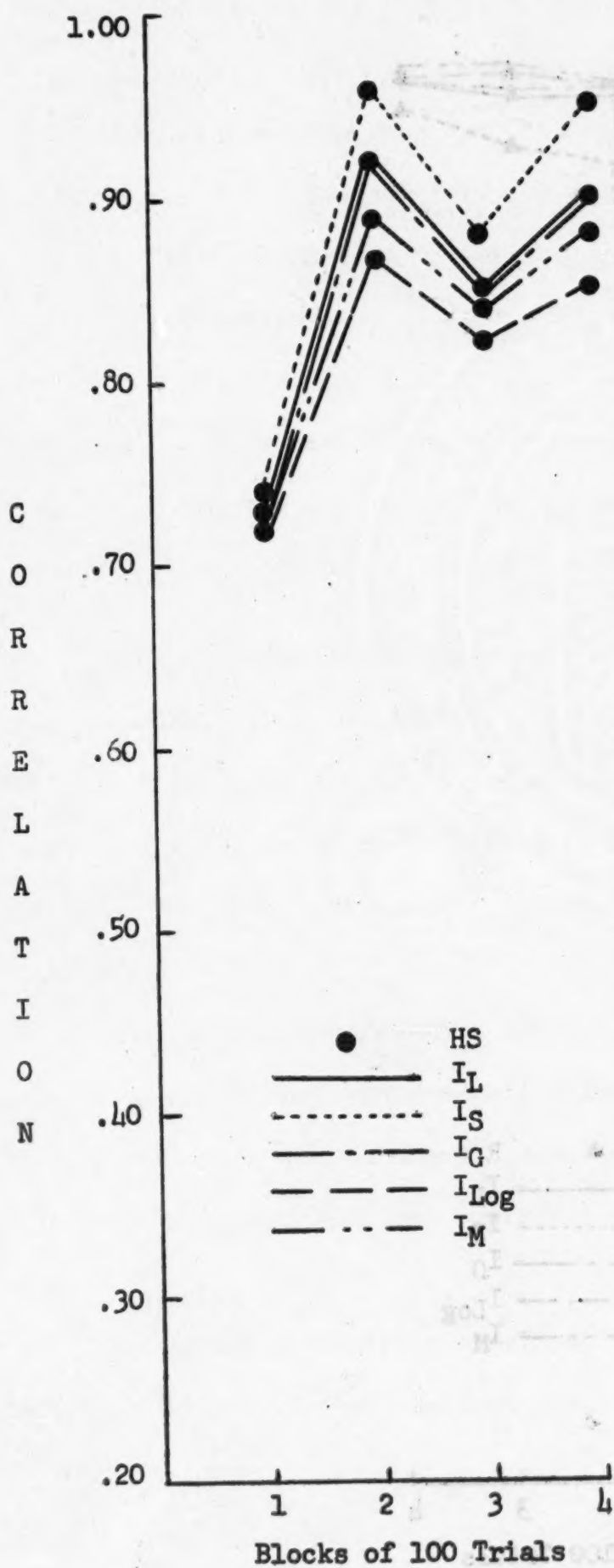


Figure 79. Correlations of the Generalization H for the Grade School Group with Linear, Squared, Geometric, Logarithmic, and Modified Log Integration methods.

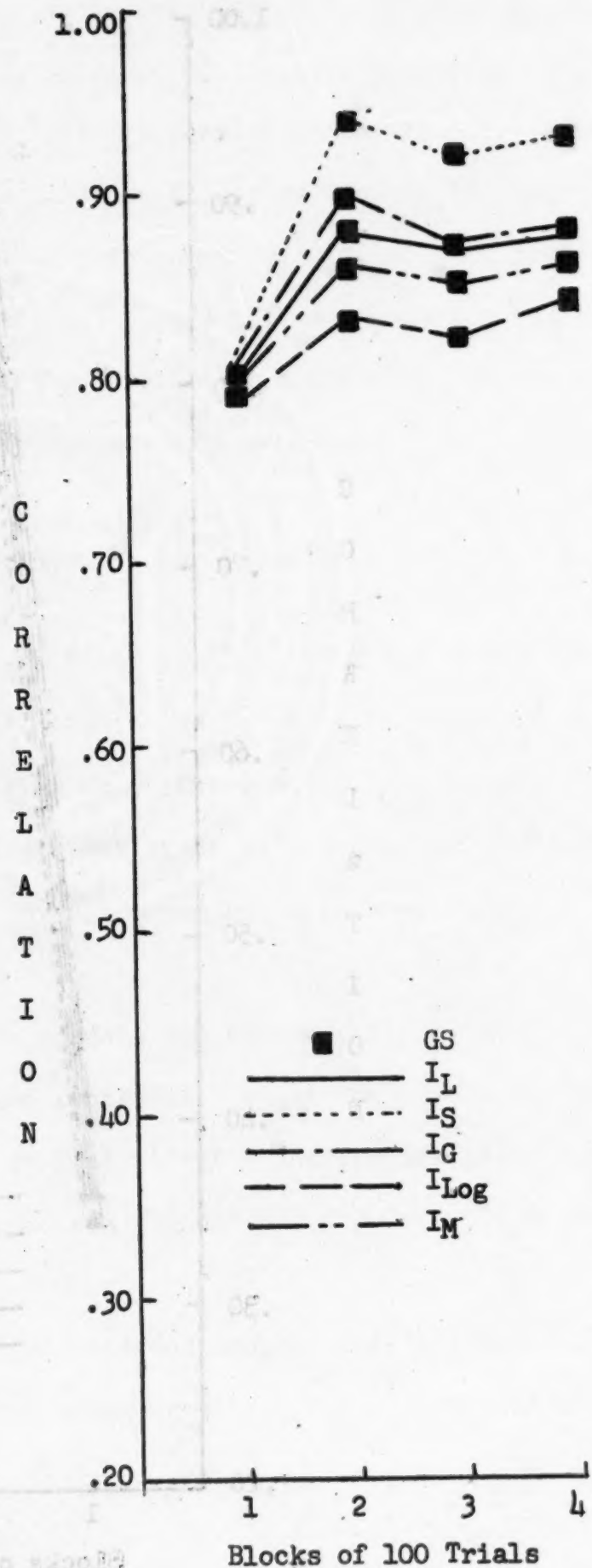
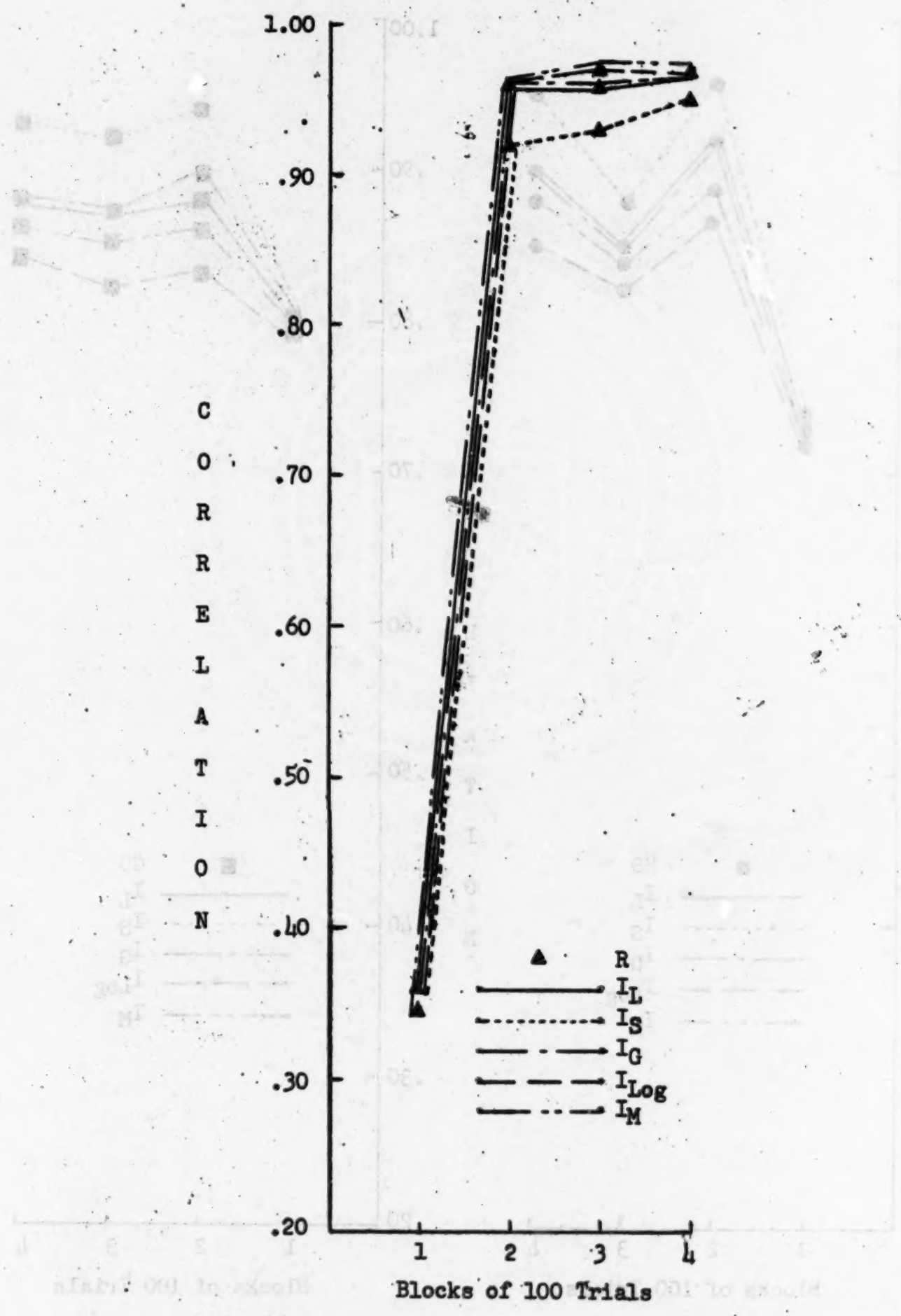


Figure 80. Correlations of the Generalization H for the Retarded Group with Linear, Squared, Geometric, Logarithmic, and Modified Log Integration methods.



With the retarded these differences are not apparent. Indeed there is some evidence (see Figure 68) that just the opposite is true for these subjects.

In Figures 72 through 80 comparison is obtained of the five synthesis formulas. The differences here are often small but are consistent particularly with the normals. For these subjects the Square Solution (I_S) provides the best prediction. This is followed by the Linear (I_L), the Geometric (I_G), the Modified Log (I_M), and the Log (I_{Log}) in the order stated. These differences are significant at the 99% level of confidence. The retarded are less consistent but there is reason to believe that the order is reversed (see Figures 74 and 80).

DISCUSSION

There is only partial agreement between these results and those of the foregoing studies. This is probably attributable to the differences in procedure and analysis. It would not be proper at this time to register an opinion concerning which is the better method. Certainly the present study is the more intricate and involves the fewest assumptions, but this may not prove to be an unmitigated blessing.

Again we see a difference between the normals and the mentally retarded. This is obtained for both generalization and synthesis. While the models are better predictors of the subnormal behavior, we find that the normals are using more efficient generalization and synthesis procedures (G_H and I_S) than are the retarded (G_S and I_{Log}).

The better predictions obtained for the retarded are probably (at least in part) due to one of the invariant analytical procedures. It can only be determined

by additional analysis, but the difference may be due to the fact that the habit growth formula used in these analyses is a better description of retarded than of normal learning processes.

STUDY NO. 4

SYNTHESIS USING QUANTITATIVE RESPONSES AND DICHOTOMOUS KNOWLEDGE OF RESULTS

The purpose of this study was to begin the development of measurement techniques which would be superior to those previously used. It was also intended as a check on the results obtained in Study No. 3.

In order to test the validity of a different approach it was considered desirable to use only subjects of unimpeachable mental ability. It is reasoned that if the technique is sound it can later be translated into a procedure acceptable to those subjects whose mental attainments are more modest, but that if the initial work were done on the retarded, for example, it would be difficult to determine how much of the variance incurred was due to the subjects' inability to synthesize as against how much was due to any defect inherent to the method. For this reason college students were used as subjects.

In Studies 1 and 3 the subjects responded X or Y and knowledge of results was also binary (correct or incorrect). In Study No. 2 quantitative responses were required and quantitative knowledge of results given, but this was at the expense of limiting the usefulness of the data to the evaluation of previously specified hypotheses. It is believed that at the present level of our understanding of these processes it is preferred to obtain data in a form that can be applied to practically any conceptual model or analytical procedure. In the present study knowledge of results was given in binary form (correct or incorrect), but the responses were scalar or quantitative.

METHOD

This study is actually an account of two separate treatments. They are included under one title since their purpose and rationale are similar. In one treatment (Scaling Response) subjects were asked to register how "X-like or Y-like" the picture was. In the second treatment (Wager Response) the subjects were allowed to place a wager on whether the picture was an X or a Y. It was assumed that the amount of wager would be proportional to the subject's degree of conviction that the picture belonged to the indicated category.

Subjects

Seventy-four college students obtained from the T. C. U. campus were tested in this experiment. Fifty were assigned to the Scaling Response Treatment and 24 to the Wager Response Treatment.

Procedure

The 400 pictures of the Selected Stimulus Group described in Study 1 were also used in this study. For the Scaling Response Group these pictures were sequentially projected on a screen, and the subjects were required to respond by first identifying the picture as either an X or Y and then quantifying it on a 10 unit scale of the dimension selected. This in effect produced a 20 unit scale ($X_{10} X_9 \text{---} X_1 Y_1 Y_2 \text{---} Y_{10}$ which could be converted into $X_1 X_2 \text{---} X_{10} X_{11} X_{12} \text{---} X_{20}$). After the response was recorded the subject was told whether the picture responded to was actually an X or a Y.

A special apparatus was constructed for testing the Wager Group. These subjects faced a translucent screen, a voltmeter (marked off with reinforcement units won or lost), a potentiometer (marked off in amount of wager), and an apparatus

energizing switch. A trial was run by first projecting the picture on the translucent screen. The subjects responded by making a bet using the potentiometer. The center position of this control was null, that is, it indicated neither an X or a Y judgment and a zero bet. The subject was forbidden to use this exact position in registering a response. Turning the knob clockwise from the center position indicated an X judgment. Turning it to the counterclockwise position from the center indicated a Y judgment. The degree of rotation from the center in either the X or Y direction determined the amount the subject was willing to wager that his judgment was correct. If he was absolutely certain that the picture was an X he was likely to turn the knob to the extreme clockwise position. An equal certainty that it was a Y would produce the extreme counterclockwise rotation. In a state of uncertainty the knob would be rotated in the desired direction but to a lesser degree. After adjusting the potentiometer knob to the desired position the subject would then press the energizing switch, and the amount of money won or lost would appear on the voltmeter.

If the subject turned the knob in the correct direction from the null position he would win an amount proportional to the degree of turn from center. If the knob was turned in the incorrect direction he would lose an amount proportional to the degree of turn from the null position. It was, consequently, to the subject's advantage to turn the knob to an extreme position when certain of the classification of the stimulus but to a lesser extent when the stimulus cues were ambiguous.

Since it was recognized on the basis of past experimentation that subjects (especially males) tend to overbet (wager not proportional to degree of conviction that the picture belongs to the chosen category) when the ratio of wins to losses

is one to one, a procedure was instituted to vary this ratio. Six of the subjects were paid on a one to one basis. This meant that for a given response the subject would win as much if the response was correct as he would lose if it was incorrect. The second group of subjects worked on a one to two ratio. They stood to win only one-half as much as they stood to lose for a given response. A third group of six subjects were put on a one to three ratio of wins to losses while the fourth group received a one to four ratio.

The subjects' responses were put on a 0-100 unit scale. Zero corresponded to the extreme counterclockwise position (strong belief that the picture was a Y) of the potentiometer while 100 represented the extreme clockwise position (strong belief that the picture was an X). The scale was linear with respect to angular position.

RESULTS

The data from each of the two treatments were applied against two of the generalization hypotheses (G_L and G_H) and three of the synthesis formulas (I_S , I_L , and I_{Log}) used in Study 3. This, in effect, produced six theoretical models for comparison. The analytical manipulations were conducted precisely as described for the earlier study except that the empirical data were the means of the subjects' quantitative responses rather than the frequency ratios previously used.

The correlations obtained between the results of the Wager Treatment and the six probability models (2 generalization functions $\times$ 3 synthesis formulas) are shown in Table 14. These correlations are higher than those obtained for

the normal subjects of Study No. 3 indicating some improvement in methodology. The differences between analytical treatments are small but are unusually consistent. This is shown in Figures 81 through 85. For purposes of illustration the correlations were ranked from highest to lowest for each comparison of each block of trials. In Figure 81 it is shown that for Generalization Solution G_H the Squared Synthesis Method I_S consistently produces the best prediction. This is followed by the Linear Synthesis Method I_L . The Log Method I_{Log} produces the poorest prediction for each block of trials.

In Figure 82, I_S is again superior to the other methods of synthesis when the Generalization Solution G_L is used. There is no consistency in the order of magnitude of the correlations of the other synthesis formulas.

In Figures 83, 84, and 85 it is shown that G_H is consistently better than G_L for all of the synthesis methods.

In recognition of these general effects it is not surprising that the model based upon I_S and G_L in combination with one another consistently produces the highest of the correlations obtained.

Turning now to the Scaling Response Treatment the results are very similar. The correlations are shown in Table 15 and are illustrated as ranked scores in Figures 86 through 90.

These are even higher than the correlations obtained using the Wager Method (.724 to .995) indicating a general superiority of the Scaling Method over the Wager Method as a procedural technique.

The comparisons of the synthesis formulas by ranking are not consistent in their results. Generally I_S produces the better prediction, but some anomalies are in evidence as will be appreciated from Figure 87. There is consistency in

Table 14

Correlations Obtained Between the Means of the Subjects'
Wagering Responses and the Six Theoretical Models

Blocks	Linear		Square		Log	
	G_H	G_L	G_H	G_L	G_H	G_L
1	.878	.745	.881	.764	.874	.741
2	.946	.931	.952	.932	.943	.930
3	.934	.882	.939	.924	.933	.920
4	.962	.949	.966	.951	.960	.949

Table 15

Correlations Obtained Between the Means of the Subjects'
Scaling Responses and the Six Theoretical Models

Blocks	Linear		Square		Log	
	G_H	G_L	G_H	G_L	G_H	G_L
1	.912	.724	.916	.798	.910	.780
2	.975	.973	.987	.973	.983	.972
3	.974	.900	.975	.925	.973	.965
4	.995	.993	.990	.984	.989	.980

Figure 81. Rank order of correlations of 24 Wager subjects for Generalization H with Linear, Squared, and Logarithmic Integration methods.

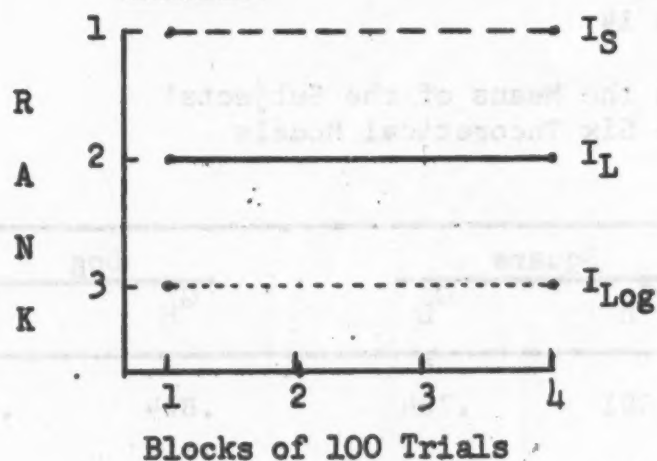


Figure 82. Rank order of correlations of 24 Wager subjects for Generalization L with Linear, Squared, and Logarithmic Integration methods.

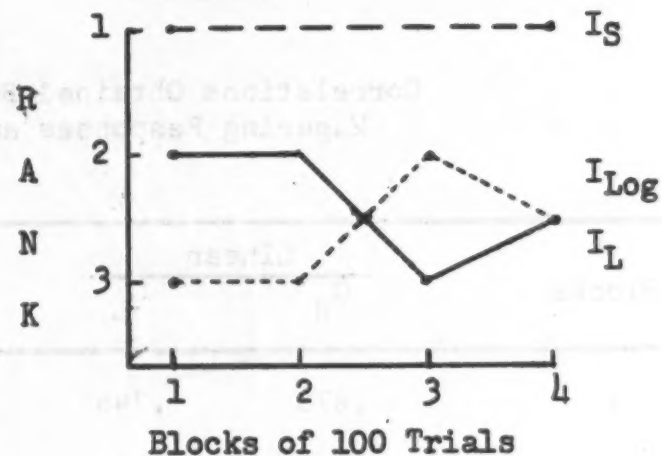


Figure 83. Rank order of correlations of 24 Wager subjects for Generalizations L and H with the Linear Integration method.

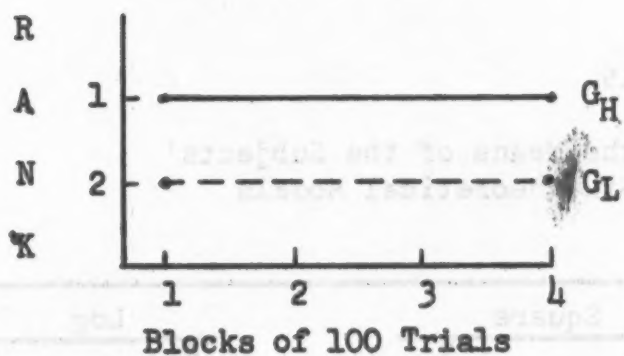


Figure 84. Rank order of correlations of 24 Wager subjects for Generalizations L and H with the Squared Integration method.

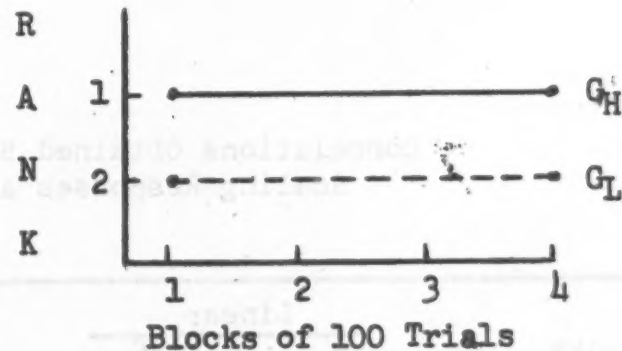


Figure 85. Rank order of correlations of 24 Wager subjects for Generalizations L and H with the Logarithmic Integration method.

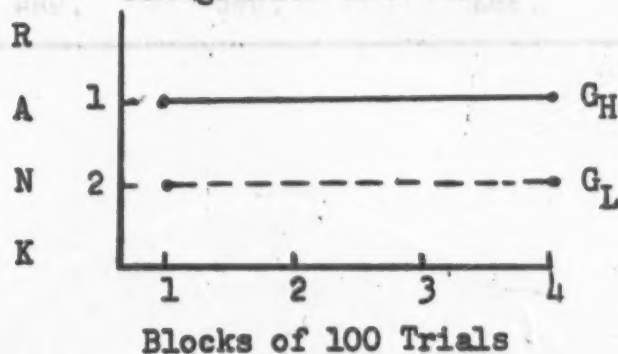


Figure 86. Rank order of correlations of 50 Scaling subjects for Generalization H with Linear, Squared, and Logarithmic Integration methods.

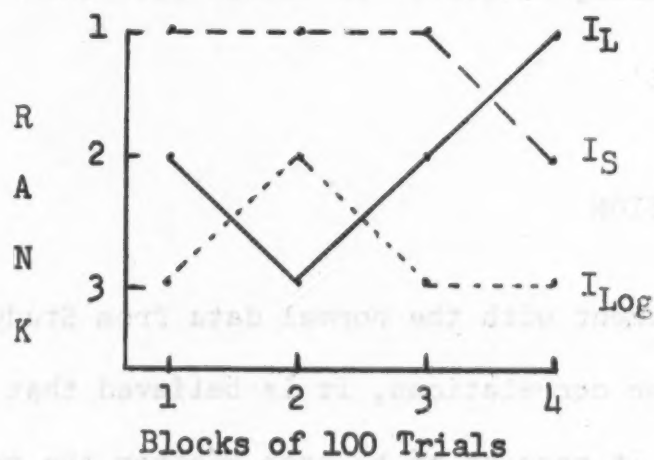


Figure 87. Rank order of correlations of 50 Scaling subjects for Generalization L with Linear, Squared, and Logarithmic Integration methods.

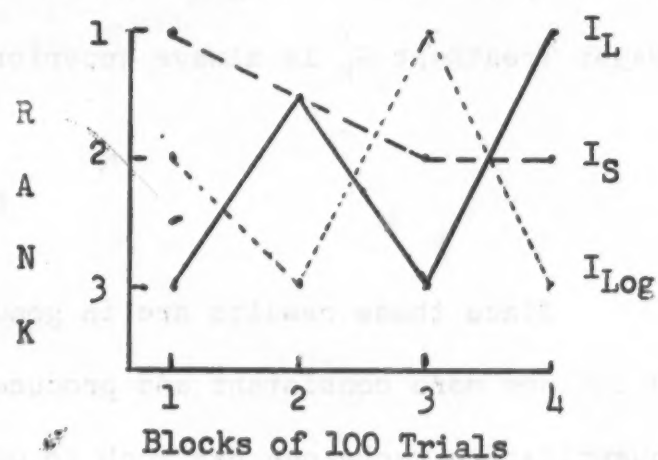


Figure 88. Rank order of correlations of 50 Scaling subjects for Generalizations L and H with the Linear Integration method.

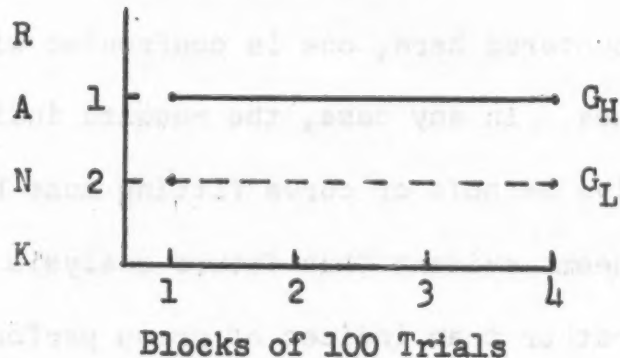


Figure 89. Rank order of correlations of 50 Scaling subjects for Generalizations L and H with the Squared Integration method.

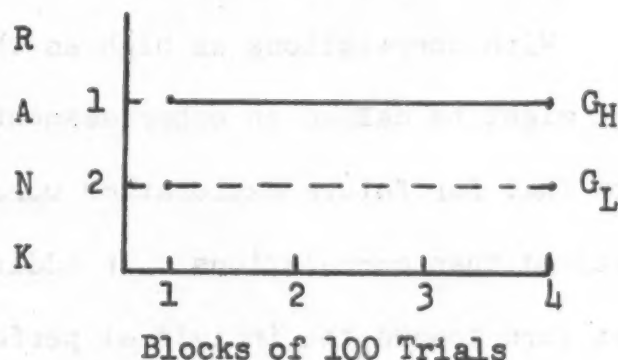
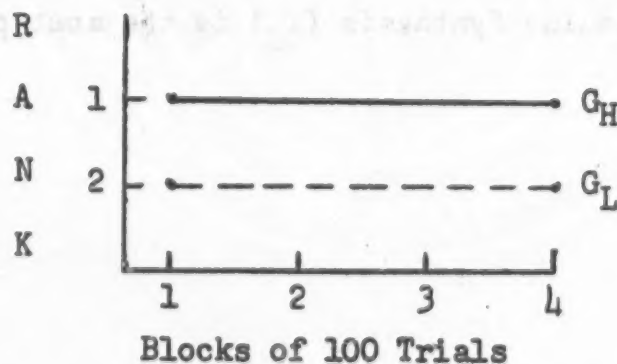


Figure 90. Rank order of correlations of 50 Scaling subjects for Generalizations L and H with the Logarithmic Integration method.



the comparisons of the generalization formulas, however. In common with the Wager Treatment G_H is always superior to G_L .

DISCUSSION

Since these results are in good agreement with the normal data from Study No. 3 but are more consistent and produce higher correlations, it is believed that the quantitative technique has much to offer. It remains to be seen whether the procedural concepts can be translated into a form that will be appropriate for testing the mentally retarded. In any case the agreement between this experiment and those previously reported encourages the belief that either the quantitative or the frequency ratio system of analysis is valid.

With correlations as high as those encountered here, one is confronted with what might be called an embarrassment of riches. In any case, the results indicate that for future exploration more sensitive methods of curve fitting must be obtained than correlations. In addition it seems evident that future analysis must turn toward the individual performance rather than indices of group performance.

In summary of this study it can be said that for normal subjects the negatively accelerated generalization function (G_H) is most descriptive of their behavior. With only slightly less certainty one can also conclude that the Squared Solution for Stimulus Synthesis (I_S) is the most precise thus far obtained.

GENERAL SUMMARY

This report includes the results of four experiments on stimulus synthesis. These studies were conducted with the view of determining whether the process differs with degree of intelligence. For this purpose, evaluation was made of both normal and retarded subjects and their performance was examined in relationship to selected hypotheses or theoretical models.

Although intelligence level and stimulus integration was of cardinal interest in these investigations, analysis was also, of necessity, made of stimulus generalization and rate of learning. In addition consideration was given to questions of procedure and data collection techniques.

In Study No. 1 comparison was made of high school, grade school, and mentally retarded subjects. The general procedure was to show the subjects a series of pictures in which four stimulus cues (each having a probability distribution assigned to their variations) denoted the response class or category to which each picture belonged. Half of the pictures belonged to response category X and the remaining to response category Y. Since there was an objective association between the variations of each cue and response category membership it was possible for the subjects to discriminate between the pictures.

In order to develop some idea of how the subjects were synthesizing the separate cues it was necessary to obtain an approximation of the degree of response to each individual cue. This was done in two ways. For the Single Stimulus Method four "control" groups were run. Each of these groups was given one of the four cues as relevant (having the same probability distribution as was obtained in the stimuli given the "experimental" groups). The remaining cues were

irrelevant (probability distribution of these cues constant at .50 throughout its variations). The results (ratio of subjects responding to the picture as an X) was then to be used as an estimate of the strength of that cue variation on that particular trial.

The second procedure (Constant Stimulus Method) of estimating the strength of the individual cue variations was by giving each subject of the "experimental" groups an additional series of pictures in which each cue was systematically varied while the remaining cues remained constant at the null position (that variation having an objective probability of .50 of belonging to either the X or Y response categories).

These estimates were applied to three different stimulus synthesis models. One of these models (the Multiplicative Solution) was based on the assumption that stimulus integration occurred in a mathematically logical fashion. Thus it was assumed that the probability of a subject responding to a picture as an X was equal to the objective probability of its being an X.

The second model (Additive Solution) envisioned the subjects as combining the individual cue probabilities in a linear fashion.

The third model (Square Root Solution) assumed that the subjects would over-emphasize the midrange of probability values.

These solutions were applied using the estimates provided by both the Single Stimulus Criteria and the Constant Stimulus Criteria.

The results indicate that in the case of normals the Additive Solution provided the best description, but that for the retarded subjects the Multiplicative Solution was most precise.

In the second study a change in procedure was effected. This study was based on the assumption that the subject behaves in a manner analogous to a closed loop control circuit. If the return signal (knowledge of results) is produced in a manner similar to the innate synthesis process (outgoing signal) then stability and good performance should result. On the other hand if the input and output channels obey different functions instability and poor performance will occur.

Three models were tested. These were (1) the Additive Solution (described above), (2) the Square Solution (similar but not so extreme as the Multiplicative Solution described above), and (3) the Log Solution which was a less extreme substitute for the Square Root Solution described above.

In this study, using normal subjects, the Additive Solution proved superior to the Square Solution which in turn was superior to the Log Solution.

In the third study a return was made to the procedure used in the first study except that the individual cue estimates were completely theoretical and were not based on any data taken from control groups or other estimates from empirical sources.

This was done by assuming that the increment of growth in the association of a cue variation with a response is inversely related to the momentary probability of its eliciting that response. In addition three stimulus generalization modes were evaluated. One (Generalization S) involved a positively accelerated decay function, the second (Generalization H) a negatively accelerated decay function, and the third (Generalization L) a linear decrement.

Five stimulus synthesis models were also evaluated. These were (1) the Additive Solution, (2) the Squared Solution, and (3) the Log Solution. These are

described above. In addition (4) a Geometric Solution, with properties between the Squared and Additive Solutions, as well as (5) a Modified Log Solution, with properties between the Log and Linear Solutions, were tested.

For normals these results favored Generalization H and the Square Solution for stimulus synthesis. The results for the retarded were less definite but appeared to be in greatest correspondence with Generalization S and the Log Solution to stimulus synthesis.

In the final study the subjects made quantitative responses. For one treatment they judged how X-like or Y-like the pictures were. In a second treatment they were allowed to bet on whether the pictures were Xs or Ys. Since the amount of the wager was optional it was assumed that it was a function of the degree of perceptual certainty.

As in the third study the results were evaluated using various theoretical models for generalization and stimulus synthesis.

This study produced the best correlation between predicted and actual results. This was especially true for the treatment in which the subjects judged each picture on the X-like and Y-like continua.

Concluding, it can be said that the results of the third and fourth studies were in very good agreement. There is agreement between the first and second studies, but there is some contradiction between these and the latter studies. In Studies 1 and 2 the Additive Solution was generally superior for the normal subjects. This was not true in Studies 3 and 4. It is too early to say whether this apparent discrepancy is due to procedure or to method of analysis. It should be remembered, however, that the latter experiments were given a more detailed analysis and appear to be more reliable.

Considering the studies as an aggregate the following conclusions are tentatively submitted.

1. The superiority of normal subjects over retarded is due in part to the difference in stimulus generalization. This is seen in the fact that the best description for the former was Generalization H but for the latter was Generalization S. As experience gathers Generalization H produces a more efficient type of discrimination than does Generalization S.

2. There is some fundamental difference in the synthesis process used by the two subject sources. All of the data testify that a difference exists, but the direction of the difference is not clear since the separate studies are not in absolute agreement. Neither group used a mathematically logical approach to the problem, but the normal group is somewhat nearer to it. The best explanation seems to be that the normal subjects begin in an efficient manner but when they learn that their solution will not yield certainty they begin to cast about for other hypotheses. There is evidence that probability relations are basically irrational to the human organism. He can accept qualifications and contingency relationships but not probability statements. He needs must structure his universe with such concepts as "B follows A" and he will not equivocate with the fundamental simplicity of this association. If B follows A only 85% of the time he immediately concludes that either A is a conglomerate made up of A' A" etc., or that B is similarly constituted. In short, he insists upon an absolutely orderly universe. He is not satisfied with a system that pays off 85% of the time. In renewing his search for a perfect solution he samples from many irrelevant hypotheses. It is possible that the retarded operate in the same manner but with slower development.

3. The assumption made in Studies 3 and 4 concerning growth rate (inversely related to momentary probability) is more descriptive of retarded than of normal behavior. This is a matter requiring additional study.

4. The quantitative response - binary knowledge of results procedure appears to be the most successful of the techniques described here. It yet remains to be seen, however, whether this experimental paradigm can be translated into a form acceptable to the mentally retarded.

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